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STRUCTURAL PARTICULARS OF GLUED LAMINATED BEAMS OF VARIABLE HEIGHT

The first part of the paper presents the main structural characteristics of laminated timber and describes some Eurocode 5 code requirements which can be more elaborate than for solid timber. The second part of the paper presents a parametric comparison of two most typical forms of double beams of variable height with levelled or saddled lower edge. The stress conditions were compared for a number of differently-shaped double-tapered and pitched cambered beams. The results obtained for a refined mesh of finite elements in the SAP2000 computer program were compared with the results obtained by simplified formulae given in Eurocode 5. It was demonstrated that, in general, a correctly modelled finite element model could account for all structural particulars of beams of variable height; while the main advantage of the finite element approach seems to be the freedom of forms, shapes, and material characteristics used in the mathematical model.

Keywords: glued laminated timber, double-tapered beams, pitched cambered beams, apex area, radial stress, stress perpendicular to the grain

Introduction

Modern glued laminated structural timber is a product of the most advanced technologies which include the most recent findings on materials, design and the theory of structures. Glued laminated timber is manufactured by gluing together individual pieces of dimension timber under controlled conditions, forming an interesting, attractive and versatile architectural and structural building material. Because it is a manufactured product, it is possible to order glulam parts for a wide variety of applications and/or configurations. Naturally, occurring defects such as knots, wanes, and checks in larger-sized timber can be controlled or eliminated in glulam by using laminations that contain only acceptable flaws. The reasons for

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the relatively small use of laminated timber structures in modern architecture and building practice are hidden not only among structural or architectural drawbacks, but sometimes also in human nature which still regards wood as temporary, flammable and unworthy of respect structural material [Stungo 2001; Kitek Kuzman 2008; Ratajczak et al. 2006, 2009].

One of the first reported tests of glulam beams of variable height was performed back in mid 1970s. For example, in [Gopu, Goodman 1975] as well as in [Gutkowski et al. 1982] the results of the first full-scale tests on (double-) tapered and curved glulam beams were reported; while in [Möhler 1976; Gopu, Goodman 1977; Möhler, Blumer 1978] the first design recommendations and simplified formulae for (double-) tapered and curved glulam beams were proposed. It was soon realised that for beams with pronounced slopes, the radial stress capacities might be problematic for practical design of pitched and tapered glued laminated beams [Gutkowski, Devey, 1984; Buckner, Gupu 1988; Götz 1996; Žagar 2002]. For a more rational design the use of additional radial reinforcement was first proposed by [Vincent, Gopu 1985] as well as by [Vonroth, Lober 1987]. The possibilities of using an alternative radial reinforcement also were addressed in more recent publications. For instance, the possibilities of using composite materials and carbon fibre reinforced polymers strips as reinforcement of glued laminated wood beams were for example examined by [Kasal, Heiduschke 2004] and [Johnsson et al. 2007] as well as by [Jasieńko et al. 2010]. The use of self-tapping screws to increase the load carrying capacity of curved beams was investigated by [Jonsson 2005]. One of the first contributions to finite elements analysis of double-tapered glulam beams can be found in [Kechter, Gutkowski 1984]; while the first computational determination of the radial reinforcement of pitch cambered beams can be found in [Vonroth, Lober 1987]. A comprehensive bibliographical review of the finite element methods (FEMs) applied in the analysis of wood products and structures can be also found in recent paper by [Mackerle 2005].

In the modern sustainability oriented architectural practice the use of various forms of beams and frames of variable height made of glued laminated wood has been becoming more and more popular [CNDB 2002; BCIT 2006; APA 2009]. The simplified formulae included in the codes can be used only for simply supported beams with pre-determined shapes. In all other cases the FEM models are widely used by structural engineers; while a number of modern computer programs facilitate more accurate modelling, analysis and more economical use of materials.

The aim of this paper is to present the appropriateness of modern general purpose structural analysis FEM computer program (e.g. SAP2000 2008) for the analysis of glulam beams of variable height. In the first part of the paper some structural properties of glued laminated timber are briefly presented. In the second part a parametric study of a series of double beams of variable height with levelled or saddled lower edge was performed. The main aspects observed were stresses in the apex area as well as maximal bending stress and its location. The results obtained for a very refined mesh of finite elements in the SAP2000 structural analysis program were compared with the formulae for simple beams given in EN 1995-1-1:2004 (Eurocode 5). It was demonstrated that, in general, a cautiously modelled finite element model, with appropriate grid density and orientation of local axes in the direction of grains, using the appropriate wood orthotropic material, can account for all structural particulars of beams of variable height. However, the main advantage of the finite element approach still is the general freedom of forms, shapes, various grain orientations, and different material characteristics used in the mathematical model.

Standards and codes for glued laminated timber

The procedures for design, fabrication, quality control and construction of glued laminated beams were standardised and included in the Eurocode standards. This paper is limited only to technical regulation related to the design of glued laminated beams. Glued laminated wood facilitates the use of structural beams of different shapes and forms. Curved members of virtually any practical radius are possible to obtain simply by forming individual pieces into desired shape prior to gluing them. Glulam members are available in much longer lengths and sizes than standard sawn timber. Beside straight beams of constant height, Eurocode 5 (Part 1-1) also includes beams of variable height in three different typical shapes:

a) Single or double-Tapered beams – fig. 1a,
b) Curved beams – fig. 1b,
c) Pitched cambered beams (Saddled beams) – fig. 1c.

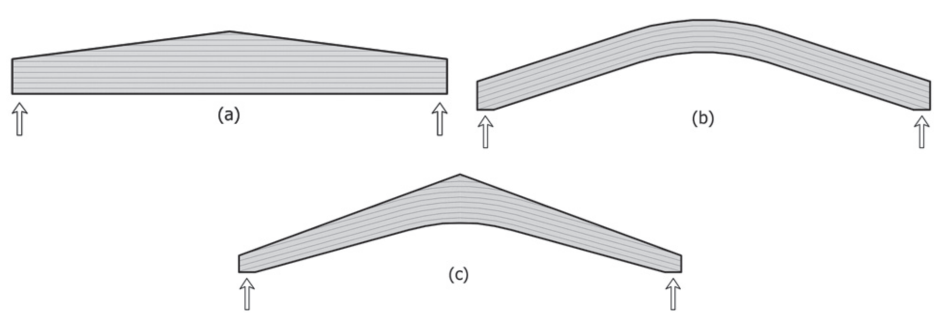


Fig. 1. Typical glued laminated beams: (a) double-tapered beam; (b) curved beam and (c) pitched cambered beam

Rys. 1. Typowe belki z drewna klejonego warstwowo: (a) belka dwutrapezowa; (b) belka zakrzywiona; (c) belka zakrzywiona o zmiennym przekroju

EN 14080 standard (Timber structures – Glued laminated timber – Requirements) contains various structural requirements for glued timber in detail. For example, it prescribes the maximum allowable thickness of laminations regarding the humidity of the environment and the radius of curvature for curved elements. Especially important is EN 1194 standard (Timber structures – Glued laminated timber – Strength classes and determination of characteristic values). For instance, an attempt at determination of the quality of grown in Poland pine timber has recently been made by [Noskowiak et al. 2010].

Fig. 2 and 3 present the various strengths for solid timber class C24 (selected for comparison) and homogenous glued laminated timber classes GL (last 2 digits denote bending strength in MPa). The values of modulus of elasticity *E* along the grain and perpendicular to the grain are presented as well. It can be seen that bending strengths for solid timber C24 and glulam GL24 are equal, but all other strengths are greater for glulam timber. The only exception is tension perpendicular to the grain which is equally small for all classes.

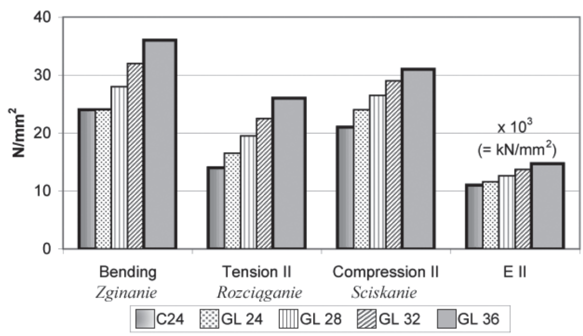


Fig. 2. Bending, tensile and compression strengths and modulus of elasticity for solid timber C24 and glulam timber
Rys. 2. Wytrzymałość na zginanie, rozciąganie i ściskanie oraz moduł elastyczności dla drewna litego C24 i drewna glulam

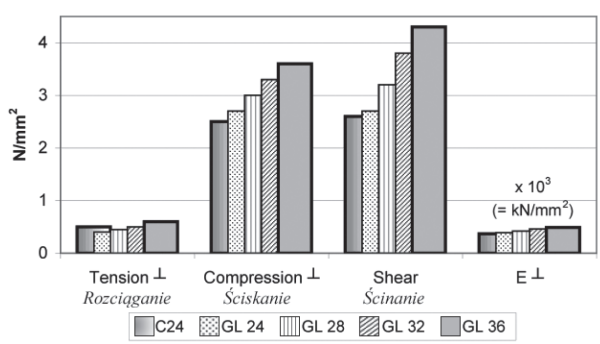


Fig. 3. Axial (perpendicular to the grain) and shearing strengths, modulus of elasticity for solid timber C24 and glulam timber
Rys. 3. Wytrzymałość osiowa (prostopadła do przebiegu włókien) i na ścinanie oraz moduł elastyczności dla drewna litego C24 i drewna glulam

In the limit state method, the structure or its component (element, cross section) is considered appropriate for use until it exceeds the limit state when the strength or usability criteria are no longer fulfilled. The expressions that define different limit criteria include separate partial safety factors for actions and for the material. The condition for a limit state design of a structure, which requires that the design value of the effect of actions (E_d) on a component is smaller than or equal to its design value of the corresponding resistance (R_d), can be written as:

$$E_d \leq R_d = R(X_d\, a_d ...) \quad \text{or} \quad E_d/R_d \leq 1 \qquad (1)$$

where: X_d is a design value of the material property,
a_d is a design value of geometrical data.

Since Eurocode 5 assumes a linear relation between stresses and deformations, the expression (1) can be further simplified and only the maximum values of design stress caused by actions (σ_d) is compared with the design values of material strength (f_d). Design values (X_d) are obtained from the characteristic values (X_k) as follows:

$$X_d = k_{mod} \cdot X_k / \gamma_m \qquad (2)$$

where: k_{mod} is a modification factor taking into account the effect of the duration of load and moisture content,
γ_m is a partial factor for the material property.

All these code requirements also were included in our analysis of selected beams of variable height.

Material and methods

General

The calculation of stresses for elements made of glued laminated timber is usually more complicated than for the elements made of other materials [Götz 1996; Žagar 2002]. The main reason for it is the variable geometry of the cross sections, inclinations of element edge regarding the grain direction in laminations and the curvature of the element axis. For double-tapered beams with varying cross-section, curved and pitched-cambered beams, special stress conditions in the apex area should be taken into account (fig. 4a, 4b). In addition to normal bending stresses σ_m, transversal-radial stresses $\sigma_{t,90}$, acting in the direction perpendicular to the longitudinal direction of beams, should also be expected. These radial stresses are in many cases the crucial parameter that determines the size of the beam in the apex area, because the design strength in the direction perpendicular to the grain is much smaller as the strength in the direction of the grain (fig. 2, 3).

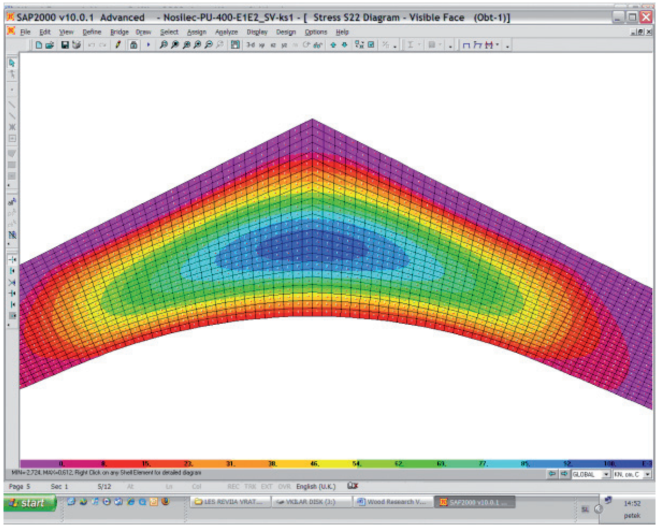


Fig. 4a. Part of the pitched cambered (saddled) beam – tensile radial stresses in the apex area in SAP2000
Rys. 4a. Część belki zakrzywionej o zmiennym przekroju (siodłowej) – naprężenia rozciągające poprzeczne w strefie kalenicy – program SAP2000

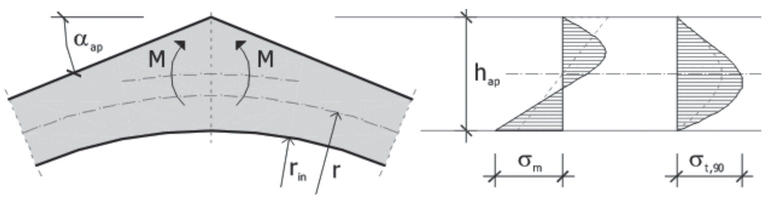


Fig. 4b. Special stress conditions in the apex area of a pitched-cambered beam. (σ_m – bending stresses, $\sigma_{t,90}$ – radial stresses; tension perpendicular to the grain direction)
Rys. 4b. Specjalne warunki naprężeń w strefie kalenicy belki zakrzywionej o zmiennym przekroju (σ_m – naprężenia zginające, $\sigma_{t,90}$ – naprężenia poprzeczne; rozciąganie prostopadłe do przebiegu włókien)

Various researchers, especially Möhler [1976], Gopu and Goodman [1977], and Möhler and Blumer [1978], had done elaborate analyses of such beams and studied the effects of the above-mentioned parameters on the beam stress conditions. They had also proposed some expressions which facilitate a simplified analytical calculation of such beams, which were later included in many national codes and standards (also in Eurocode 5 – see Chapter 4.2). In codes, these specific stress conditions are usually considered by different correction factors for the calculation of bending and radial stresses. The most accurate analyses were done by solving the differential equation of a wall made of orthotropic material. Several experimental research studies confirmed the results [Gopu, Goodman 1975; Gutkowski et al. 1982]. Today it is possible to assess more complicated stress conditions of such beams, also by using a finite element computer program (e.g. SAP2000 or other). In this case, the mathematical model should be prepared with care in order to take into account adequate numerical accuracy as well as different material properties in different directions corresponding to actual grain directions in glulam elements.

Parametric study

This paper presents a comparative analysis of two most typical shape representatives of beams of variable height, i.e. double-tapered beams and pitched-cambered beams (also called saddled beams). It is assumed that the beams are supported only by vertical reactions and that the overturning or buckling out of their plane is prevented [Eilering, Halbensleben 2007]. The geometry data was derived from a straight beam made of GL28h with a constant rectangular cross section designed for a given span (16 m) and assumed design uniform load (21 kN/m). According to Eurocode 5, the height of such a beam according to bending criteria should be exactly h = 100 cm, if beam's width is fixed to b = 20 cm. The frontal area of such a beam amounts to 16 m^2 and the volume to 3.20 m^3. Seven variants of double--tapered and seven variants of saddled beams (all with $2c$ = 0.25 and L = 4 m, see fig. 6b) considered in the study had the same volume as the original straight beam. For this reason, their basic geometry data (h_{ap}, α_{ap} and r_{in} – see fig. 6a) was modified individually for each case as shown in table 1. To ease the comparisons, the individual beams were labelled with one letter (T for tapered and S for saddled) and the roof slope in percent (= 100·tan α_{ap}). Because the same wood volume was used to produce the beams, the slopes of saddled beams were higher as the slopes of tapered beams. All modified geometry parameters are presented in table 1.

Table 1. Modified geometry data for considered beams
Tabela 1. Zmodyfikowane dane geometryczne dla rozważanych belek

Double-Tapered beams *Belki dwutrapezowe*				Pitched cambered beams (Saddled beams) *Belki zakrzywione o zmiennym przekroju (siodłowe)*			
SLOPE [%] *Nachylenie [%]*	h_{ap} [cm]	α_{ap} [°]		SLOPE [%] *Nachylenie [%]*	h_{ap} [cm]	α_{ap} [°]	r_{in} [cm]
T-0	100	0.00		S-12.5	114.2	7.13	1732
T-2.5	110	1.43		S-15.0	117.3	8.53	1457
T-5.0	120	2.86		S-18.75	122.2	10.62	1182
T-7.5	130	4.29		S-25.0	130.8	14.04	910
T-10.0	140	5.71		S-37.5	149.8	20.56	643
T-12.5	150	7.13		S-50.0	171.0	26.57	513
T-15.0	160	8.53		S-62.5	194.1	32.01	438

Mathematical modelling

The glued laminated beams were modelled as orthotropic 4-node final elements with appropriate orientation of material characteristics in different directions. Each (symmetric) beam half was divided into 25 elements in height and 100 elements in length (all together 5,000 elements were used for each beam). The size of each element was roughly 4 by 8 cm with an approximate size ratio 1:2. The elements used were not equal due to variable geometry, but their local material axes were always oriented in the direction of grain of the actual material in a glulam member (Direction 1 – parallel to the grain, Direction 2 – perpendicular to the grain). The glulam was modelled as orthotropic material of different characteristics in two different directions. The modulus of elasticity E amounted to 1260 kN/cm^2 for Direction 1 and to 42 kN/cm^2 for Direction 2. The corresponding shear modulus amounted to 78 kN/cm^2 for all directions. In tapered beams, Direction 1 was always parallel to global X direction and the modelling was not problematic. More demanding was the modelling of pitched cambered beam, where Direction 1 local axis was always perpendicular to the radius of curvature of each element. In this case, there was a need to use cylindrical coordinates to ease the modelling of the finite elements mesh. The mesh of finite elements used together with the direction of the local axes for analysed shape variants are presented in fig. 5.

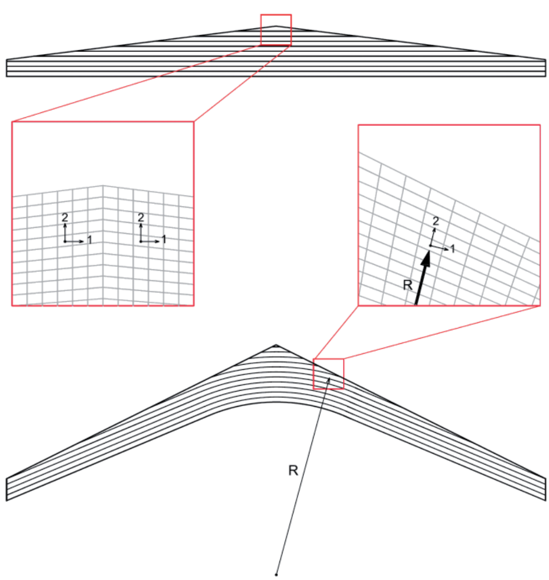


Fig. 5. Orientation of local axes of finite elements in the direction of the grain in actual glulam member and the finite elements mesh in SAP2000

Rys. 5. Kierunek ustawienia lokalnych osi elementów skończonych zgodny z kierunkiem przebiegu włókien w prawdziwej części belki glulam oraz siatka elementów skończonych w programie SAP2000

Additionally, such a finite element approach facilitates combination of different wood qualities (e.g. higher wood quality on lower and upper sides of the beam), as well as the inclusion of various grain orientations, material characteristics, and forms and shapes in the mathematical model.

Results

Stresses in the apex area

Computer analysis of glued laminated beams facilitates calculation of the complete stress distribution for any cross section. This is an advantage over the simplified expressions from the standards, which facilitate only stress calculations at a certain point where the maximum values are expected. fig. 6 shows selected results for analysed double-tapered beams (left) and saddled beams (right). Fig. 6a and 6b show the geometry of the analysed beams. The following four figures show the design bending stresses. Fig. 6c and 6d show the contour lines of the same bending stresses and fig. 6e and 6f the distribution of normalised bending stresses over the height of the beams in the apex area. The last four figures show the design transverse-radial stresses. Fig. 6g and 6h show the contour lines of the same radial stresses and fig. 6i and 6j the distribution of normalised radial stresses over the height of the beams in the apex area. Because the beam heights change, they are presented as relative values in the way that the "zero" coordinate means the lower edge and 1.0 coordinate the upper edge of the beam. Also, the design values of stresses were normalised corresponding to their design value of strength, which is also shown as reference value among the results in fig. 6. The positive values mean tension and the negative values compression. The design bending stresses ($\sigma_{m,d}$) are normalised with design bending strength ($f_{m,d}$). According to Eurocode 5, this value is obtained for a reference height amounting to 60 cm for glued laminated beams. For lower beams the bending strength is slightly higher (maximum up to 10%). Another important factor that should be considered is the strength reduction due to bending of the laminates during production. For these elements reduction of bending strength depends on the ratio between the radius of curvature and the thickness of laminates. However, these modification factors did not apply to the selected geometry of beams, and design bending strength for material GL28 (short-term load duration class, service class 2 (k_{mod}=0.90) and material safety factor γ_m=1.25) amounted to $f_{m,d}$ = 2.016 kN/cm^2.

Also the design transverse-radial stresses ($\sigma_{t,90,d}$) were normalised to their design strength according to Eurocode 5. The initial design tensile strength for the direction perpendicular to the direction of fibres for selected material GL28h (k_{mod}=0.90 in γ_m=1,25) amounted to $f_{t,90,d}$ = 0.032 kN/cm^2. This value should be increased due to the effect of stress distribution in the apex area (dashed part of the beam in fig. 6a). The amplification factor (k_{dis}) amounted to 1.4 for double-tapered and curved beams and to 1.7 for saddled beams. However, the design tensile strength should be reduced due to the effect of the size (volume) of the apex area V ($V_{max} = 2 \cdot V_b/3$, where V_b is total volume of the beam and $V_0 = 0.01$ m^3 is the reference volume value). The final value of design tensile strength can be expressed as:

$$f_{t,90,d}^{'} = k_{dis} \cdot \left(\frac{V_0}{V}\right)^{0.2} f_{t,90,d} \tag{3}$$

The results in the apex area show the characteristic stress distribution for normal bending and transversal radial stresses, which are consistent with the research results obtained by various authors [Möhler, Blumer 1978; Žagar 2002]. Also, the obtained stress anomaly on the bottom edge in fig. 6j is known from the method of finite elements (MKE). The correct value is zero, because there is no load on the lower beam edge. It can be seen from the distribution of stresses that the numerical results are limited toward the correct zero value on the lower beam edge.

It can be seen in fig. 6e and 6f that the bending stresses in the apex zone are not exceeded for any analysed beam variant. With the increase in beam slope, the bending stresses decrease, but the radial stresses rapidly exceed the design tensile strength (fig. 6i, 6j). The increase in the apex height, which is the consequence of changed geometry, only slightly reduces the maximal radial stress. The increase in radial stresses is not dangerous for doubled tapered beams in the analysed range, but it seems to be critical for all pitched cambered beams with slopes higher than approximately 10%. In the cases where the slope exceeds 50%, the actual radial stress is already almost five times greater than the allowable design strength. The main problem is that the design tensile strength in the direction perpendicular to fibre direction is low. In the discussed case, the only solution is to increase dimensions of the beam in the apex area. Therefore, it can be concluded that for a given span and load, the increase in saddled beam slope or the increase in beam curvature requires an additional increase in beam volume, which is actually unfavourable from the structural and economic points of view.

Another solution exists for the beams characterised by high transversal-radial stresses. Special reinforcing devices can be used which prevent splitting of lamellas in radial directions. These can be made of wood, metal [Vonroth, Lober 1987] or even of fibre reinforced plastic [Kasal, Heiduschke 2004]. Wooden reinforcing devices are usually made of harder wood plates which can be glued on both sides of the beam [Jonsson 2005]. Additionally, nails can be used in order to increase pressure during the gluing process. The metal devices are usually internal steel screws for wood or ribbed steel bars. Epoxy resin should be poured in pre-drilled holes before mounting transversal screws.

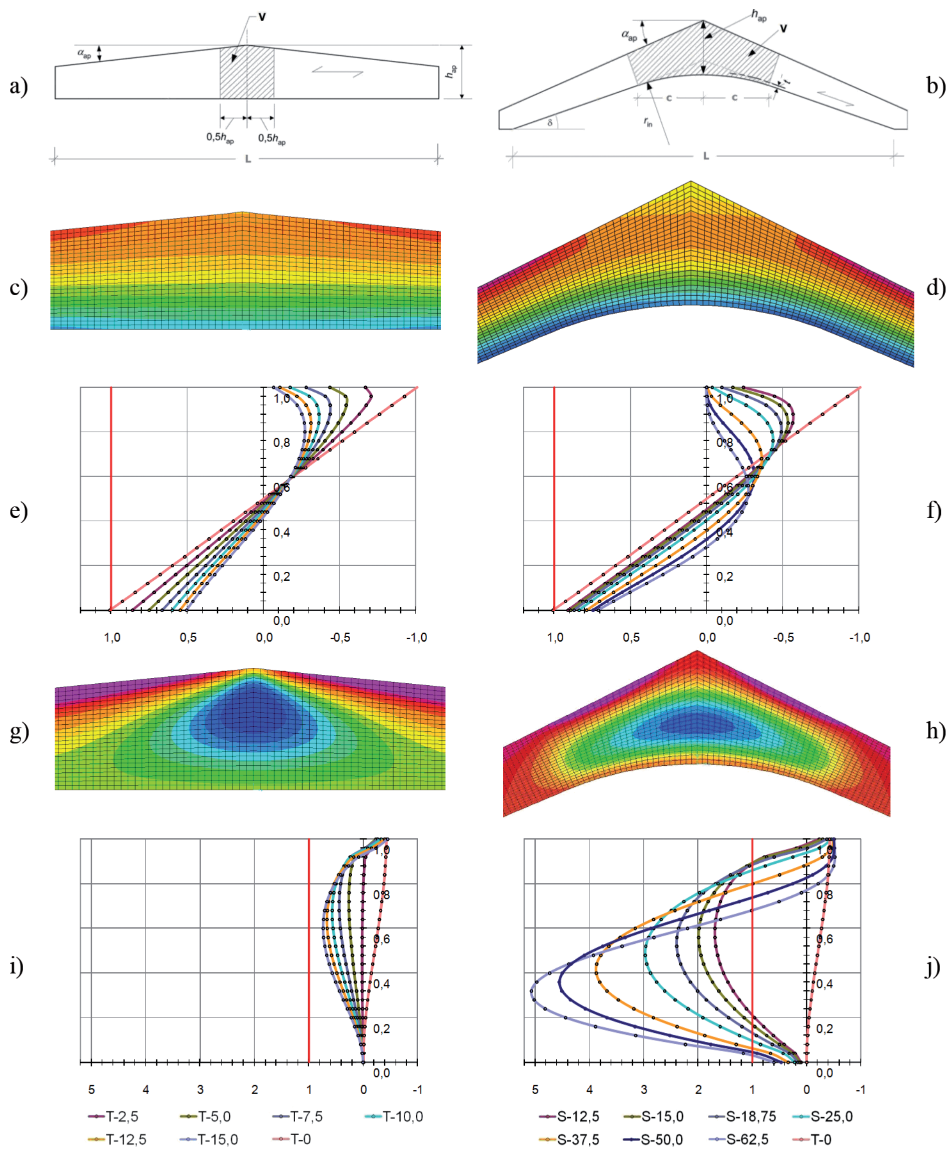


Fig. 6. (a) Double-Tapered beams and (b) Pitched cambered beams (Saddled beams); (c, d) Contours of the normal – bending stresses; (e, f) Distribution of the normal – bending stresses over the height of the beam in the apex area; (g, h) Contours of the transversal – radial stresses; (i, j) Distribution of the transversal – radial stresses over the height of the beam in the apex area

Rys. 6. (a) Belki dwutrapezowe i (b) belki zakrzywione o zmiennym przekroju (siodłowe); (c, d) Kontury prostopadłej – naprężenia zginające; (e, f) Rozkład prostopadłej – naprężenia zginające na wysokości belki w strefie kalenicy; (g, h) Kontury linii poprzecznej – naprężenia poprzeczne; (i, j) Rozkład linii poprzecznej – naprężenia poprzeczne na wysokości belki w strefie kalenicy

Comparison with Eurocode 5

The simplified formulae in Eurocode 5 for calculation of maximal design bending stresses are based on the assumption of linear correlation between deformations and stresses. The more complicated stress conditions in the beams with different shapes are taken into account with correction factors that are used to multiply the previously-obtained bending stresses. For all types of considered beams an increase of bending stresses in the apex area should be considered. In the apex area, transverse-radial stresses are usually the most critical. In the simplified calculation approach, they are determined as a percentage of normal stresses depending on the inclination of the upper edges of the beam and/or on the curvature of the lamellas. The radial stresses are then compared with design tensile strength in the direction perpendicular to the wood fibres, corrected with two factors that include the effect of stress distribution and actual volume of the apex area [Larsen 2001]. Therefore, the actual stress condition is obtained using different correction factors (k_s on the load side and k_f on the capacity side). The stress condition for bending and for radial stresses can be symbolically expressed as:

$$\sigma'_{\_d} = k_\sigma \cdot \sigma_{\_d} \le f'_{\_d} = k_f \cdot f_{\_d} \qquad (4)$$

The equations for correction factors for different shapes of beams of variable height are given in Eurocode 5 (Chapter 6.4). The applied expressions and their 3D graphical representations are briefly summarised in Appendix 1.

The comparison between the values obtained by SAP program and the values obtained by Eurocode 5 using the simplified expressions and correction factors described above, are presented in fig. 7. It can be seen that the correlation is fairly good and that the simplified procedure from the Eurocode is on the safe side.

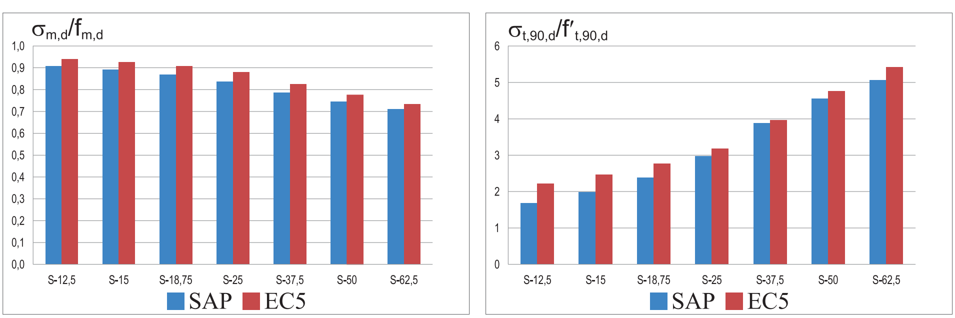


Fig. 7. The comparison of normalised stresses (left bar chart: normal bending stresses; right bar chart: radial stresses) obtained for the apex area of the analysed pitched cambered beams using SAP2000 and Eurocode 5

Rys. 7. Porównanie znormalizowanych naprężeń (lewy wykres: normalne naprężenia zginające; wykres prawy: naprężenia poprzeczne) otrzymane dla strefy kalenicy analizowanych belek zakrzywionych o zmiennym przekroju przy zastosowaniu programu SAP2000 i kodeksu Eurocode 5

Beside the stress conditions in the apex area, another important beam design parameter is the maximal bending stress which does not necessarily occur in the apex area, but rather at the location where the ratio between internal bending moment and beam inertia moment reaches its maximal value. According to elastic beam bending theory (EBT) the design bending stresses $\sigma_{m,d}(x)$ can be simply determined as the quotient of design bending moment $M_d(x)$ and section modulus $W(x)$ of the rectangular cross section with variable beam height $h(x)$. In double tapered beams and in straight parts of saddled beams the height change is linear and it depends on angles α_{ap} and d, as well as the height at the support h_s. However, in the apex area of saddled beams the height of the beam does not change linearly anymore and additionally it depends on the length of the curved apex area c, which, together with angle d, determines the radius of curvature of the lower beam edge in this area. The $h(x)$ can be most rationally expressed if the coordinate system's origin is placed in the middle of the beam. The expressions obtained for saddled beams are as follows:

- in the apex area where the change of beam height is nonlinear (x < c):

$$h(x) = h_s + \left(\frac{L}{2} - x\right)\tan\alpha_{ap} - \frac{L}{2}\tan\delta + \frac{2c}{\sin(2\delta)} - \sqrt{\left(\frac{c}{\sin\delta}\right)^2 - x^2} \tag{5a}$$

- in the "near support" area where the change of beam height is linear (c ≤ x ≤ L/2):

$$h(x) = h_s + \left(\frac{L}{2} - x\right)\left(\tan\alpha_{ap} - \tan\delta\right) \tag{5b}$$

Fig. 8 presents obtained normalised values of bending stresses obtained using "Elastic Bending Theory (EBT)". Only one (right) half of the beam is presented. Note that $x/L=0$ stands for the middle beam span and $x/L=0.5$ for the end of the beam near the support.

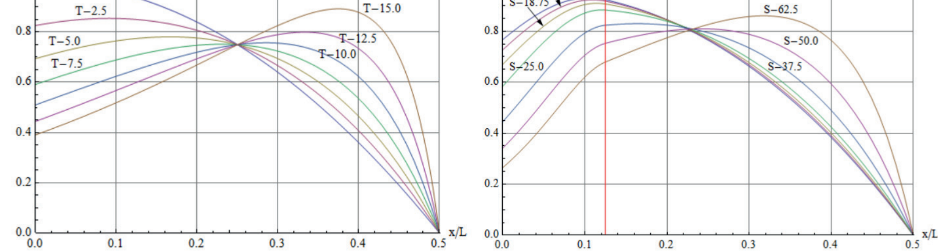
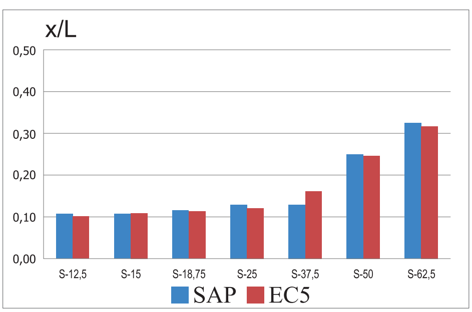


Fig. 8. Normalised bending stresses in double tapered (left) and saddled (right) beams
Rys. 8. Znormalizowane naprężenia zginające w belkach dwutrapezowych (z lewej) i w belkach siodłowych (z prawej)

It can be seen that the position of maximal stress varies significantly with the beam slope. When the beam slope is small, the maximal stress can be expected closer to the mid span and when the slope is more significant, the maximal stress occurs closer to the support. This holds true for both types of examined beams. Only for smaller slopes of examined saddled beams the maximum stress have actually occurred in the curved apex area ($x/L < 0.125$ or $x < c = 2$ m – to the left from the red vertical line in fig. 8).

For saddled beams a comparison of results obtained by EBT theory and program SAP was made as well. In fig. 9 the coordinates of maximal stress and obtained maximal values for different saddled beams are compared. In most cases the agreement is satisfying. In some cases the position of maximal stress in SAP is difficult to obtain, because very similar values appear in different finite elements. However, the maximal values never differ for more than 10%. In the cases with very high slopes, the simple linear beam theory might have reached its limits. The whole procedure of calculating maximal stresses using Equations 5a and 5b and their coordinates for EBT is rather complicated and was processed in Mathematica [Wolfram 2003].



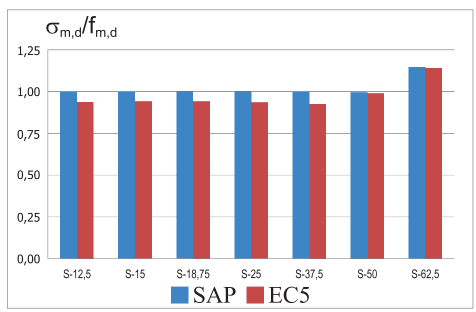


Fig. 9. The comparison of the position of maximal stress in (left) and normalised values of maximal bending stresses (right) for analysed pitched cambered beams obtained using SAP2000 and the Elastic Bending Theory (EBT)

Rys. 9. Porównanie umiejscowienia maksymalnego naprężenia w analizowanych belkach zakrzywionych o zmiennym przekroju (z lewej) i charakteryzujących je znormalizowanych wartości maksymalnych naprężeń zginających (z prawej) otrzymanych przy zastosowaniu programu SAP2000 oraz Teorii Zginania Plastycznego (EBT)

In this case the use of SAP demonstrates some significant advantages over EBT, because it facilitates calculation of the complete stress distribution for any cross section. However, in this case the maximum stress is obtained for individual finite elements instead of being obtained for a specific cross section. This is an advantage over the simplified expressions from the standards which only facilitate stress calculation at certain points where the maximum values are expected.

Discussion and conclusions

This paper demonstrates that structural characteristics and code required methods for analysis of glued laminated beams are much more elaborate than in the case of solid timber. The most important difference consists in better controlled mechanical characteristics defined for the reference glulam element height and depending on numerous parameters such as material quality, loading duration, moisture, usability class, residual stresses, as well as on the element size, the radius of curvature and the thickness of lamellas.

Even more elaborate is the behaviour of glued laminated beams of variable height. In many cases, the radial stresses constitute the crucial parameter that determines dimensions of such beams in the apex area, because the design strength of wood in the direction perpendicular to the grain is always much smaller than the strength in the direction of the grain. In this analysis, a series of beams of different heights and curvatures was selected and the results obtained by SAP program and simplified formulae from Eurocode 5 were compared. Beside normal bending stresses σ_m, and radial stresses $\sigma_{t,90}$, in the apex area, maximal bending stresses and their locations were observed as well. The comparison of the results indicates that the correlation is fairly good for all examined cases.

From the results obtained for the beams of the same volumes, but of different shapes, it can be concluded that while the slope of the beams increases, the bending stresses decrease and the radial stresses increase. The increase in radial stresses was never critical for any of the analysed double-tapered beams (slope £ 15%); however, for the pitched cambered beams the situation in the apex area became critical for all slopes greater than approximately 10%. The radial stresses in those cases considerably exceeded the design tensile strength perpendicular to fibre direction. The increase in the apex height, which is the consequence of the changed geometry, only slightly reduces the maximal radial stress. Therefore, for pitched cambered beams with the slope greater than 10%, the only solution is to increase the dimensions of the beam in the apex area or to accommodate the undesirable radial stresses with additional anchors or other measures. The situation could also be improved by adding a tensile element connecting the supports, which would minimise horizontal displacement and reduce radial stresses in the apex area. In general, it can be concluded that for a given span and load, the increase in the beam slope is favourable only for tapered beams. For pitched cambered beams, the increase in the beam slope and curvature over certain value requires an additional increase in the beam volume, which is actually unfavourable from the structural and economic points of view.

It was also confirmed that correctly and cautiously prepared finite element model can describe all particular stress conditions which are important for practical design of glulam beams of variable height. The finite element analysis facilitates calculation of the complete stress distribution for any cross section and any form of a beam or similar frame type structure. Additionally, such a finite element approach facilitates combination of different wood qualities (e.g. higher wood quality on the lower and upper sides of the beam), as well as the inclusion of various grain orientations, material characteristics and shapes in the mathematical model. This is an advantage over the simplified expressions from the standards, which facilitate only stress calculation at certain points where the maximum values are expected. To obtain the correct results, however, the mathematical model and finite element mesh should be prepared with great care and adequate precision in order to take into account satisfying numerical accuracy as well as realistic orthotropic material characteristics in different directions corresponding to the actual grain directions in tapered pitched cambered or curved glulam members.

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Appendix 1

For beams of variable height, Eurocode 5 requires the fulfilment of the stress condition which can be symbolically expressed as:

$$\sigma'_{\_d} = k_\sigma \cdot \sigma_{\_d} \leq f'_{\_d} = k_f \cdot f_{\_d} \tag{6}$$

where: coefficient k_s summarises the product of correction factors on the demand side,
coefficient k_f summarises the product of other correction coefficients, which should be applied on the capacity side.

Bending stresses

For double-tapered and pitched cambered beams considered in this analysis, the demand side correction coefficient k_s equals coefficient k_l in Eurocode 5, which can be calculated as:

$$k_\sigma = k_\ell = k_1 + k_2 \cdot \left(\frac{h}{r}\right) + k_3 \cdot \left(\frac{h}{r}\right)^2 + k_4 \cdot \left(\frac{h}{r}\right)^3 \tag{7}$$

$$k_1 = 1 + 1.4 \tan\alpha + 5.4 \tan^2\alpha$$

$$k_2 = 0.35 - 8 \tan\alpha$$

$$k_3 = 0.6 + 8.3 \tan\alpha - 7.8 \tan^2\alpha$$

$$k_4 = 6 \tan^2\alpha$$

where: h is the maximal height of the beam in the apex area (h ^{o} h_{ap}),
a is the beam slope,
$r = r_{in} + 0.5\ h_{ap}$.

For double-tapered beams, the h/r ratio equals zero and $\boldsymbol{k_\ell} = k_1$. The relation between a, h/r and coefficient $\boldsymbol{k_\ell}$ is presented in fig. 10.

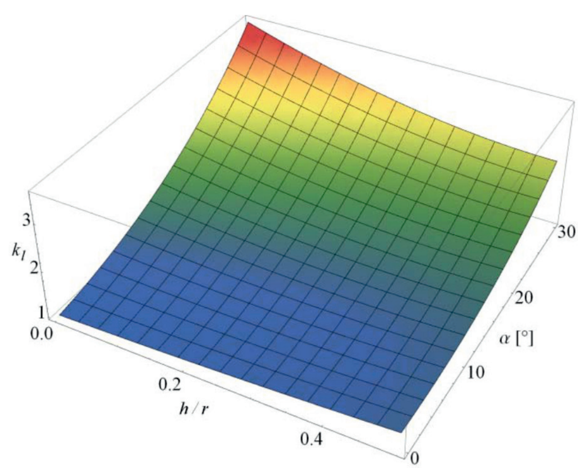

Fig. 10. Relation between *a*, *h/r* and coefficient $\boldsymbol{k_\ell}$
Rys. 10. Zależność pomiędzy a, h/r i współczynnikiem $\boldsymbol{k_\ell}$

The capacity side correction coefficient k_f includes the reduction of bending strength due to beam curvature (factor k_r) and the correction due to the element size (factor k_h):

$$k_r = \begin{cases} 1 & \cdots \quad r_{in}/t \geq 240 \\ 0.76 + 0.001\,(r_{in}/t) & \cdots \quad r_{in}/t < 240 \end{cases} \tag{8}$$

$$k_h = \min.\begin{cases} (60/h)^{0.1} \\ 1.1 \end{cases} \tag{9}$$

where: r_{in} is the internal curvature radius of the lower beam edge,
't' is the thickness of the laminates.

The correction factor k_h should be used for all elements loaded with bending or tension that are lower than 60 cm. In this case, the product of correction coefficients for the capacity side can be expressed as:

$$k_f = k_r \cdot k_h \tag{10}$$

Radial stresses

For double-tapered and pitched cambered beams considered in this analysis, the demand side correction coefficient k_s equals coefficient k_p in Eurocode 5 which can be calculated as:

$$k_\sigma = k_p = 0.2\tan\alpha + (0.25 - 1.5\tan\alpha + 2.6\tan^2\alpha)\cdot\left(\frac{h}{r}\right) + (2.1\tan\alpha - 4\tan^2\alpha)\cdot\left(\frac{h}{r}\right)^2 \tag{11}$$

For double-tapered beams, the *h/r* ratio equals zero. The relation between *a*, *h/r* and coefficient k_p is presented in fig. 11.

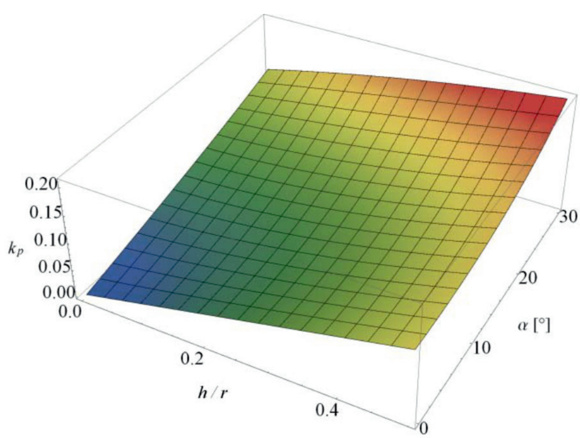

Fig. 11. Relation between a, h/r and coefficient k_p
Rys. 11. Zależność pomiędzy a, h/r i współczynnikiem k_p

The capacity side correction coefficient k_f in this case includes the reduction due to the effect of the size (volume) of the apex area V ($V_{max} = 2 \cdot V_b/3$, where V_b is the total volume of the beam and $V_0 = 0.01$ m^3 is the reference volume value) and the amplification factor ($k_{dis} = 1.4$ for double-tapered and curved beams and 1.7 for saddled beams):

$$k_f = k_{dis} \cdot \left(\frac{V_0}{V} \right)^{0.2} \quad (12)$$

SPECYFIKA STRUKTURALNA KLEJONYCH WARSTWOWO BELEK O ZMIENNEJ WYSOKOŚCI

Streszczenie

Nowoczesne klejone warstwowo drewno strukturalne jest produktem wykorzystującym najbardziej zaawansowane technologie, które czerpią z najnowszych odkryć dotyczących materiałów, projektowania i teorii struktur. Drewno klejone warstwowo jest wytwarzane przez sklejenie poszczególnych elementów drewna wymiarowego w kontrolowanych warunkach, co skutkuje powstaniem interesującego, atrakcyjnego i uniwersalnego materiału budowlanego do zastosowań architektonicznych i strukturalnych. Celem niniejszego opracowania jest sprawdzenie możliwośći stosowania nowoczesnych programów komputerowych FEM do analizy belek glulam o zmiennej wysokości. W pierwszej części artykułu zaprezentowano najważniejsze właściwości strukturalne drewna klejonego i opisano niektóre wymagania kodeksu Eurocode 5. W drugiej części artykułu przedstawiono porównanie parametryczne dwóch najbardziej typowych form belek podwójnych o zmiennej wysokości z wyrównaną lub siodłową krawędzią dolną. Porównano warunki naprężeń belek dwutrapezowych i zakrzywionych o zmiennym przekroju i różnych kształtach. Obserwowano naprężenie w strefie kalenicy, jak również maksymalne naprężenie zginające oraz jego lokalizacje. Wyniki uzyskane dla oczyszczonej siatki elementów skończonych w programie komputerowym SAP2000 porównano z wynikami uzyskanymi przy zastosowaniu uproszczonych formuł podanych w kodeksie Eurocode 5. Udowodniono, że model dla elementów skończonych, o odpowiedniej gęstości siatki współrzędnych i z lokalnymi osiami ustawionymi w kierunku przebiegu włókien, wykorzystujący odpowiedni materiał drzewny, może wskazywać wszystkie szczegóły strukturalne belek o zmiennej wysokości. Jednakże, główną zaletą podejścia opartego na wykorzystaniu elementów skończonych pozostaje ogólna swoboda wyboru form, kształtów, różnych orientacji włókien i różnych specyfikacji materiałów zastosowanych w modelu matematycznym.

Słowa kluczowe: drewno klejone warstwowo, belki dwutrapezowe, belki zakrzywione o zmiennym przekroju, strefa kalenicy, naprężenie poprzeczne