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COMPARISON OF ANISOTROPIC STRENGTH CRITERIA IN THE BIAXIAL STRESS STATE

In our study, three phenomenological strength criteria (the von Mises criterion, the Tsai-Wu criterion, and the Ashkenazi criterion) were compared based on experimental data. The goal of this study was to decide which strength theory describes properly the failure of wood under biaxial load. Formerly, a set of biaxial tests was performed on Norway spruce (Picea abies [L.] Karst.) specimens. As a result hundreds of biaxial strength data are available. Failure criteria were validated in these planar stress states. Theoretical and experimental background was presented and tensor components were calculated from engineering strength for each strength criteria. Failure prediction data was determined by the three theories. The results show that out of the three criteria the Ashkenazi strength criterion best describes the failure of wood in planar stress state.

Keywords: anisotropic strength criteria, biaxial tests, planar stress states, failure prediction number, Ashkenazi strength criterion

Introduction

At the macroscopic level physical and mechanical properties of wood and wood based materials are anisotropic because of the complex microstructure. Due to the complex structure of wood prediction of strength is difficult. Norms are limited, and the orthotropic (orthogonally anisotropic) elastic properties are not used for strength prediction.

The stress state can be linear, planar or spatial in the critical point of wooden structures. Since wood is anisotropic, strength criteria may apply to the strength of wood and wood-based materials. There are several theories in practice. Some of them were developed for the aircraft industry, because formerly some aircraft parts were built using glass-reinforced plastics or wood or wooden composites.

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These days, scientists explore this topic [Yoshihara 1991; Kasal, Leichti 2005; Mackenzie, Eberhardsteiner 2005; Seweryn, Romanovych 2007; van der Put 2009], because better mechanical design of wood needs deeper theoretical background.

A former work [Szalai 1992, 2008] compared using a theoretical approach the von Mises, the Tsai-Wu, and the Ashkenazi anisotropic strength criteria. These criteria are phenomenological criteria.

Phenomenological strength criterion applies to the phenomenon of failure only and cannot be used to explain the fracture mechanism itself. In this research the von Mises, the Tsai-Wu, and the Ashkenazi anisotropic strength criteria were compared based on their applicability. Biaxial experimental data were used [Eberhardsteiner 2002] to validate the failure criteria. Based on this strength data, anisotropic strength criteria could be compared and it could be verified which theory described the failure of wood and wood-based materials in planar stress state.

Theoretical background

The formula of the anisotropic strength criteria are most often polynomial equations. The left-hand member contains polynomials of the component of the stress tensor. The right-hand member contains scalar. The general anisotropic strength criterion has the following form [Szalai 1994]:

$$a_{ij}\sigma^{ij} + a_{ijkl}\sigma^{ij}\sigma^{kl} + a_{ijklmn}\sigma^{ij}\sigma^{kl}\sigma^{mn} + a_{ijklmnop}\sigma^{ij}\sigma^{kl}\sigma^{mn}\sigma^{op} + \ldots \le c, \qquad (1)[1]$$

with $i,j,k,l,m,n,o,p,\ldots=1,2,3$

where:

σ^{ij} – components of the stress tensor,

$a_{ij}, a_{ijkl}, a_{ijklmnop}, \ldots$ – strength tensors, range of 2,4,6,8…,

c – optional scalar.

If formula (1) is true, the material will not fracture. The anisotropic strength criteria use different part of the general strength criterion. Main anisotropy strength criteria are the von Mises criterion, the Tsai-Wu criterion, and the Ashkenazi criterion.

The strength criteria can be modelled by geometrical methods. In the case of planar stress state, the strength surface can be governed by the strength criteria.

[1] Einstein's convention was used here and hereinafter.

The von Mises strength criterion

R. von Mises [1928] applied the quadratic term of the polynomial equation (1).

$$a_{ijkl}\sigma^{ij}\sigma^{kl} \leq 1, \qquad i,j,k,l = L,\ R,T \tag{2}$$

where:

L is the longitudinal or grain direction, R is the radial direction, and T is the tangential direction within the tree stem.

Tensor components can be calculated from the engineering strength. The engineering strength was determined experimentally [Szalai 2001]. Table 1 summarises the applied engineering strength values. Tensor components in the von Mises tensor are the following:

$$a_{iiii} = \frac{1}{\left(f_i^+\right)^2} \quad \text{or} \quad = \frac{1}{\left(f_i^-\right)^2}, \qquad i = L \text{ or, } R \text{ or } T, \tag{3a}$$

where:

f_{i}^+ and f_{i}^- tension and compression strength in the anatomical main directions,

and

$$\left(a_{ijij} + a_{ijji} + a_{jiij} + a_{jiji}\right) = \frac{1}{\left(t_{ij}\right)^2}, \qquad \mathrm{i}, j = L,\ R, \text{ or } L,\ T, \text{ or } R,\ T, \tag{3b}$$

where:

t_{ij} – shear strength on the anatomical main planes.

Table 1. Engineering strength [Szalai 2001] of Norway spruce (*Picea abies* [L.] Karst) in the LR anatomical main plane

Tabela 1. Wytrzymałość inżynieryjna [Szalai 2001] świerka pospolitego (Picea abies [L.] Karst) w podłużno-promieniowej anatomicznej płaszczyźnie głównej

Spruce *Świerk*	f_L^+	f_L^-	$f_{LR}^{T(45)+}$	$f_{LR}^{T(45)-}$	f_R^+	f_R^-	t_{LR}	$t_{LR}^{T(45)+\,*}$	$t_{LR}^{T(45)-\,*}$	–
N	315	319	292	325	302	291	–	–	–	Pcs
Mean *Średnia*	63.52	49.34	9.15	9.08	5.92	3.49	9.32	4.00	4.57	MPa
CoV	23.6	18.0	28.6	25.5	28.2	22.4	43.0	28.6	25.6	%

* Data from Szalai [1994]
* *Dane od Szalai [1994]*

Other no zero tensor components are the interactive tensor components. Physical interpretation is difficult. These components could be determined by several methods. In the framework of our research the following determination methods were applied:

$$\left.\begin{aligned}
\left(a_{iijj}+a_{jjii}\right)&=\frac{4}{\left(f_{ij}^{k(45)+}\right)^2}-\frac{1}{\left(f_i^{+}\right)^2}-\frac{1}{\left(f_j^{+}\right)^2}-\frac{1}{\left(t_{ij}^{+}\right)^2}\\
\left(a_{iijj}+a_{jjii}\right)&=\frac{4}{\left(f_{ij}^{k(45)-}\right)^2}-\frac{1}{\left(f_i^{-}\right)^2}-\frac{1}{\left(f_j^{-}\right)^2}-\frac{1}{\left(t_{ij}^{+}\right)^2}\\
\left(a_{iijj}+a_{jjii}\right)&=\frac{1}{(f_i^{+})^2}+\frac{1}{(f_j^{-})^2}-\frac{1}{(t_{ij}^{k(45)+})^2},\\
\left(a_{iijj}+a_{jjii}\right)&=\frac{1}{(f_i^{-})^2}+\frac{1}{(f_j^{+})^2}-\frac{1}{(t_{ij}^{k(45)-})^2}
\end{aligned}\right\}\tag{3c}$$

$i,j = L,\ R,$ or $L,\ T,$ or $R,\ T,$

where:

$f_{ij}^{k(45)+}$, $f_{ij}^{k(45)-}$, $t_{ij}^{k(45)+}$, $t_{ij}^{k(45)-}$ – tension, compression and shear strength in the directions of the i and j axes, respectively, and in directions lying in the ij plane and making 45° angle with the i and j axes. It is very difficult to determine $t_{ij}^{k(45)+}$, and $t_{ij}^{k(45)-}$, because generating a clear shear stress state is problematic. The shear strengths in these directions determined for spruce by Szalai [1994] were used.

Graphical representation of the von Mises strength criterion is the strength surface governed by eq. 4. Shear component can be expressed from eq. 2.

$$\sigma^{ij}=\sqrt{\frac{1-a_{iiii}\sigma^{ii}\sigma^{ii}-a_{jjjj}\sigma^{jj}\sigma^{jj}-(a_{iijj}+a_{jjii})\sigma^{ii}\sigma^{jj}}{a_{ijij}+a_{ijji}+a_{jiij}+a_{jiji}}}\tag{4}$$

$i,j= L,R$ or L,T or $R,T.$

Theoretically, the surface is an ellipsoid, where the stress components coincide with the main axes of the ellipsoid. Fig. 1 shows the von Mises strength surface in the LR anatomical main plane of the Norway spruce (*Picea abies* [L.] Karst.).

The Tsai-Wu strength criterion

S.W. Tsai and E.M Wu [1971] applied the first two parts of the general strength criterion (eq. 1.). The Tsai-Wu strength criterion has the following form:

$$a_{ij}\sigma^{ij}+a_{ijkl}\sigma^{ij}\sigma^{kl}\le 1,\qquad i,j,k,l = L,\ R,\ T\,.\tag{5}$$

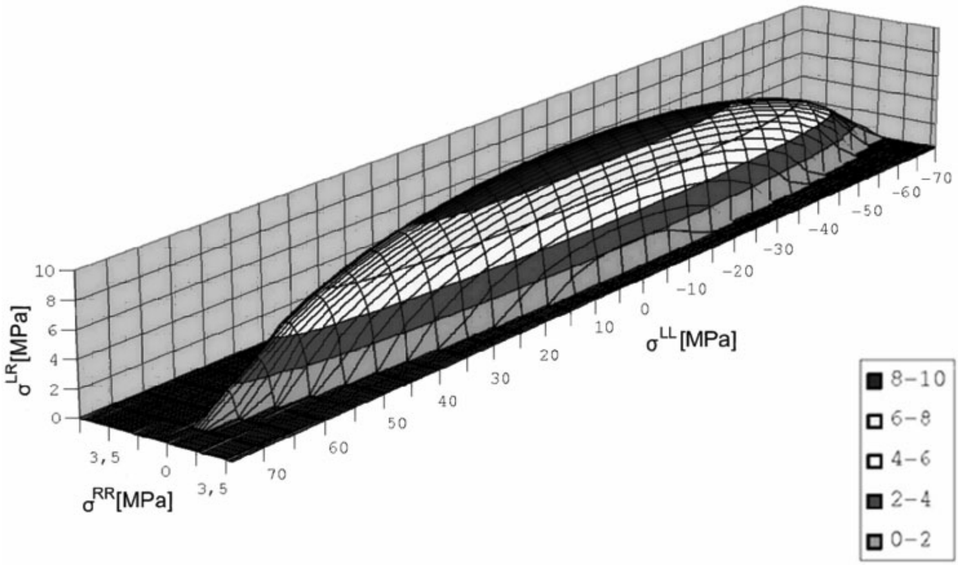

Fig. 1. Strength surface of Norway spruce (*Picea abies* [L.] Karst.) in the LR anatomical main plane governed by the von- Mises strength criterion
Rys. 1. Powierzchnia wytrzymałości świerka pospolitego (Picea abies [L.] Karst.) w podłużno-promieniowej anatomicznej płaszczyźnie głównej według kryterium wytrzymałości von Mises'a

The tensor components in the second and fourth range are:

$$a_{ii}=\frac{1}{f_i^+}-\frac{1}{f_i^-},\qquad i=L \text{ or } R \text{ or } T, \tag{6a}$$

$$a_{iiii}=\frac{1}{f_i^+ f_i^-},\qquad i=L \text{ or } R \text{ or } T, \tag{6b}$$

$$a_{ij}=\frac{1}{t_{ij}^+}-\frac{1}{t_{ij}^-}=0,\qquad i,j=L, R \text{ or } L, \mathrm{T} \text{ or } R, T, \tag{6c}$$

$$\left(a_{ijij}+a_{ijji}+a_{jiij}+a_{jiji}\right)=\frac{1}{t_{ij}^+ t_{ij}^-},\qquad i,j=L, R \text{ or } L, \mathrm{T} \text{ or } R, T \tag{6d}$$

The applied determination method for the interactive components was the following:

$$\left.\begin{aligned}
\left(a_{iijj}+a_{jjii}\right)&=\frac{4}{\left(f_{ij}^{k(45)+}\right)^2}\left[\begin{aligned}&1-\frac{f_{ij}^{k(45)+}}{2}\left(\frac{1}{f_i^+}-\frac{1}{f_i^-}+\frac{1}{f_j^+}-\frac{1}{f_j^-}\right)-\\&-\frac{\left(f_{ij}^{k(45)-}\right)^2}{4}\left(\frac{1}{f_i^+f_i^-}+\frac{1}{f_j^+f_j^-}+\frac{1}{t_{ij}^+t_{ij}^-}\right)\end{aligned}\right]\\
&\text{or,}\\
\left(a_{iijj}+a_{jjii}\right)&=\frac{4}{\left(f_{ij}^{k(45)-}\right)^2}\left[\begin{aligned}&1+\frac{f_{ij}^{k(45)-}}{2}\left(\frac{1}{f_i^+}-\frac{1}{f_i^-}+\frac{1}{f_j^+}-\frac{1}{f_j^-}\right)-\\&-\frac{\left(f_{ij}^{k(45)-}\right)^2}{4}\left(\frac{1}{f_i^+f_i^-}+\frac{1}{f_j^+f_j^-}+\frac{1}{t_{ij}^+t_{ij}^-}\right)\end{aligned}\right]
\end{aligned}\right\}\qquad(6e)$$

$$\left.\begin{aligned}
(a_{iijj}+a_{jjii})&=-\frac{1}{(t_{ij}^{k(45)+})^2}\left[\begin{aligned}&1-t_{ij}^{k(45)+}\left(\frac{1}{f_i^+}-\frac{1}{f_i^-}+\frac{1}{f_j^+}-\frac{1}{f_j^-}\right)-\\&-(t_{ij}^{k(45)+})^2\left(\frac{1}{f_i^+f_i^-}+\frac{1}{f_j^+f_j^-}\right)\end{aligned}\right]\\
&\text{or,}\\
(a_{iijj}+a_{jjii})&=-\frac{1}{(t_{ij}^{k(45)-})^2}\left[\begin{aligned}&1+t_{ij}^{k(45)-}\left(\frac{1}{f_i^+}-\frac{1}{f_i^-}+\frac{1}{f_j^+}-\frac{1}{f_j^-}\right)-\\&-(t_{ij}^{k(45)-})^2\left(\frac{1}{f_i^+f_i^-}+\frac{1}{f_j^+f_j^-}\right)\end{aligned}\right]
\end{aligned}\right\}\qquad(6f)$$

$i,j = L, R$ or L, T or R, T.

Graphical representation of the Tsai-Wu strength criterion is also an ellipsoid but the main axes of the ellipsoid and the stress coordinates are not parallel. Fig. 2 shows the Tsai-Wu strength surface in the planar stress state in the LR anatomy main plane. The expression of the strength surface in the planar stress state is the following:

$$\sigma^{ij}=\sqrt{\frac{1-a_{ii}\sigma^{ii}-a_{jj}\sigma^{jj}-a_{iiii}\sigma^{ii}\sigma^{ii}-a_{jjjj}\sigma^{jj}\sigma^{jj}-(a_{iijj}+a_{jjii})\sigma^{ii}\sigma^{jj}}{a_{ijij}+a_{ijji}+a_{jiij}+a_{jiji}}}\qquad(7)$$

$i,j = L,R$ or L,T or R,T.

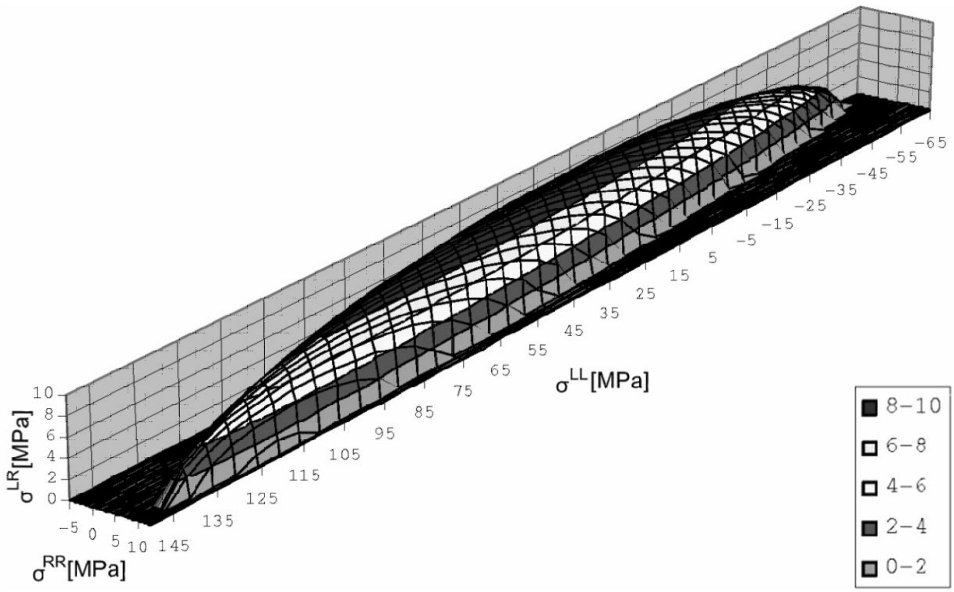

Fig. 2. Strength surface of Norway spruce (*Picea abies* [L.] Karst.) in the LR anatomical main plane governed by the Tsai-Wu strength criterion
Rys. 2. Powierzchnia wytrzymałości świerka pospolitego (Picea abies [L.] Karst.) w podłużno-promieniowej anatomicznej płaszczyźnie głównej według kryterium wytrzymałości Tsai-Wu

The Ashkenazi strength criterion

Ashkenazi [1966, 1967, 1976], Ashkenazi-Ganov [1972] defined a strength tensor in the fourth range. The structure of the tensor is similar to that of the compliance tensor. Ashkenazi applied the second and the fourth part of the general strength criterion (eq. 1.). After transformation [Szalai 1994] the following terms are applicable:

$$\frac{a_{ijkl}\sigma^{ij}\sigma^{kl}}{\left|\sqrt{I_1^2 - I_2}\right|} \le 1, \tag{8}$$

where:

I_1 and I_2 – the first and the second stress invariants.

The components of the strength tensors are as follows:

$$a_{iiii} = \frac{1}{f_i^+} \quad \text{or} \quad = \frac{1}{f_i^-}, \qquad i = L \quad \text{or} \quad R \quad \text{or} \quad T, \tag{9a}$$

$$\left(a_{ijij}+a_{ijji}+a_{jiij}+a_{jiji}\right)=\frac{1}{t_{ij}},\qquad i,j=L,R \text{ or } L,T \text{ or } R,T, \tag{9b}$$

$$\left.\begin{aligned}&\left(a_{iijj}+a_{jjii}\right)=\frac{4}{f_{ij}^{k(45)+}}-\frac{1}{f_i^{+}}-\frac{1}{f_j^{+}}-\frac{1}{t_{ij}},\\ &\text{or}\\ &\left(a_{iijj}+a_{jjii}\right)=\frac{4}{f_{ij}^{k(45)-}}-\frac{1}{f_i^{-}}-\frac{1}{f_j^{-}}-\frac{1}{t_{ij}},\end{aligned}\right\}\quad i,j=L,R \text{ or } L,T \text{ or } R,T \tag{9c}$$

and

$$\left.\begin{aligned}&\left(a_{iijj}+a_{jjii}\right)=\frac{1}{f_i^{+}}+\frac{1}{f_j^{+}}-\frac{1}{t_{ij}^{k(45)+}},\\ &\left(a_{iijj}+a_{jjii}\right)=\frac{1}{f_i^{+}}+\frac{1}{f_j^{+}}-\frac{1}{t_{ij}^{k(45)-}}\end{aligned}\right\}\quad i,j=L,R \text{ or } L,\mathrm{T} \text{ or } R,T. \tag{9d}$$

$t_{ij}^{k(45)+}, t_{ij}^{k(45)-}$ were also taken from Szalai [1994]. Eq. 8 presents a surface in the fourth range. In the planar stress state the following terms are available:

$$\sigma^{\mathrm{ij}}=\pm\sqrt{\frac{1}{q_{ij}}\left[\frac{1}{2q_{ij}}-a_{iiii}\left(\sigma^{ii}\right)^2-a_{jjjj}\left(\sigma^{jj}\right)^2-\left(a_{iijj}+a_{jjii}\right)\sigma^{ii}\sigma^{jj}\pm\right.}$$
$$\left.\pm\sqrt{\frac{1}{4q_{ij}^2}-\left(\frac{a_{iiii}}{q_{ij}}-1\right)\left(\sigma^{ii}\right)^2-\left(\frac{a_{jjjj}}{q_{ij}}-1\right)\left(\sigma^{jj}\right)^2-\left(\frac{a_{iijj}+a_{jjii}}{q_{ij}}-1\right)\sigma^{ii}\sigma^{jj}}\ \right] \tag{10}$$

where:

$$q_{ij}=a_{ijij}+a_{ijji}+a_{jiij}+a_{jiji},\qquad i,j=L,R \text{ or } L,T \text{ or } R,T.$$

Fig. 3 shows that the strength surface is the general surface. The Ashkenazi strength surface may be convex, whilst in the previous two strength criteria it may be concave. Hence, the convex surface better fits to the stress points than the concave surface.

Materials and methods

Experimental data [Eberhardsteiner 2002] were verified using three anisotropic strength criteria (the von Mises criterion, the Tsai-Wu criterion, and the Ashkenazi criterion). Since the stress states were valid in the moment of the fracture, the strength criteria should predict the failure.

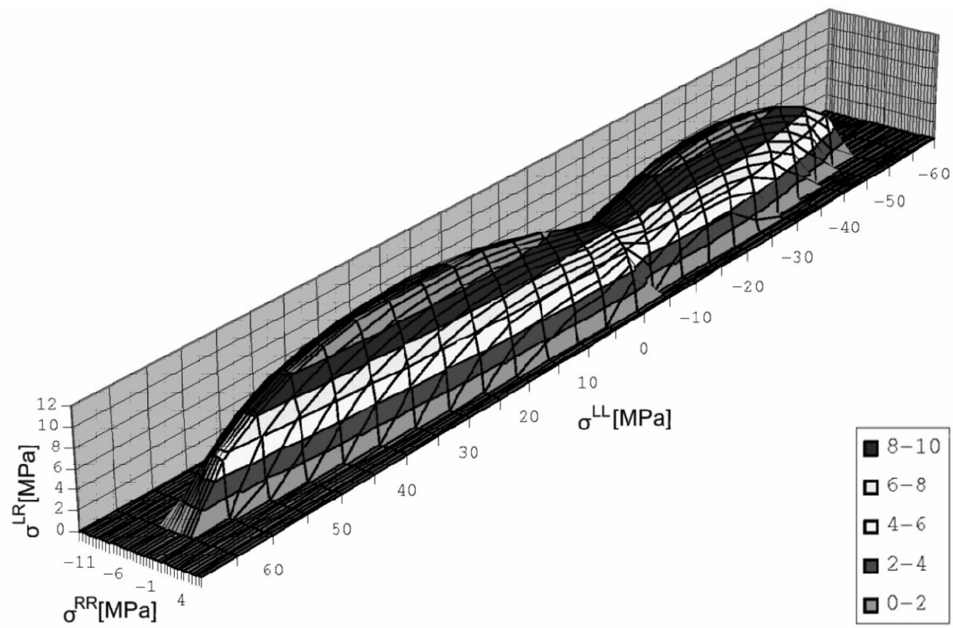

Fig. 3. Strength surface of Norway spruce (*Picea abies* [L.] Karst.) in the LR anatomical main plane governed by the Ashkenazi strength criterion
Rys. 3. Powierzchnia wytrzymałości świerka pospolitego (Picea abies [L.] Karst.) w podłużno-promieniowej anatomicznej płaszczyźnie głównej według kryterium wytrzymałości Ashkenazi'ego

Eberhardsteiner [2002] measured 423 pieces of cross-shaped Norway spruce (*Picea abies* [L.] Karst.) specimens under biaxial load. The specimens were cut out from the stem along the LR (longitudinal-radial) anatomical main plane. The estimated cross section was 140 × 140 mm^2. The thickness depends on the loading mode. Linear u and v displacements were applied along the x and y axes, according to the axes of the biaxial testing machine. The reaction was the planar stress state with σ_x, σ_y stress components. Bending was not permitted (when tension directions did not coincide with the anatomical main axes). The biaxial load was applied by displacements so that the ratio $\kappa = \bar{u}/\bar{v}$ of their mean values $\bar{u}$ and $\bar{v}$ remains constant (fig. 4).

Relatively to the axes of the specimen the grain angle *(φ)* was different. φ = 0° *(L)*, 7,5°,15°,30°, and 45°. Experimental tests were performed at the room temperature. The average moisture content of wood was 12%. Six measurements were carried out for each combination of the parameters φ and κ. Fracture was defined when extremities arose among σ_x and σ_y . After the investigations 423 stress states were available at failure.

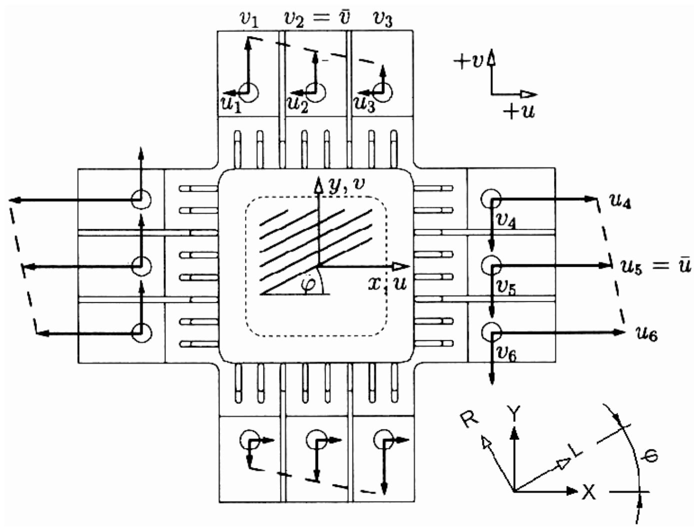

Fig. 4. Biaxial test specimen. Specimen shape, loading type and position of the coordinate systems are presented
Rys. 4. Próbka do badań dwuosiowych. Przedstawiono kształt próbki, rodzaj obciążenia oraz pozycję systemów współrzędnych

The biaxial stress state results at the fracture were correlated with the main axes of the specimens and the loading directions. The anisotropic strength criteria could not be applied because they are suitable only if the stress state is correlated to the orthotropic directions. The obtained stress states had to be transformed from the *xy* to the *LR* system. The transformation was based on the orthotropic properties and the angle *(φ)* between the anatomical main directions (fig. 4):

$$\left.\begin{aligned} \sigma^{LL} &= \sigma^{xx}\cos^2\varphi + \sigma^{yy}\sin^2\varphi \\ \sigma^{RR} &= \sigma^{xx}\sin^2\varphi + \sigma^{yy}\cos^2\varphi \\ \sigma^{LR} &= (\sigma^{xx} - \sigma^{yy})\sin\varphi\cos\varphi \\ \sigma^{RL} &= \sigma^{LR} \end{aligned}\right\} \tag{11}$$

The tensor components are different according to the stress states. The tensor components depend on the sign of the normal stresses in a given stress state. Four groups can be defined ($\sigma^{LL+}\sigma^{RR+};\sigma^{LL+}\sigma^{RR-};\sigma^{LL-}\sigma^{RR+};\sigma^{LL-}\sigma^{RR-}$).

The tensor components were determined for each strength criterion. The following formulas were applied: eq. 3a–3c for the von Mises strength criterion, eq. 6a–6f for the Tsai-Wu strength criterion, and eq. 9a–9d for the Ashkenazi strength criterion. The engineering strengths were applied based on table 1. The normal strengths were applied from Szalai [2001], and the diagonal shear strengths $t_{LR}^{T(45)+}$ $t_{LR}^{T(45)-}$ were applied from Szalai [1994] as well.

After the described procedures the strength criteria were applicable. The term of the failure prediction number had to be determined. The failure prediction number correlates to the failure. The right-hand member of eq. 1. is a scalar. If this scalar equals 1, the strength criterion will describe the failure. It is an ideal case. If the scalar is less than 1, the strength criterion will not predict any failure, but the material is damaged. If the scalar is more than 1, the theory will predict failure, but the material remains intact. The graphical meaning is the following: if the scalar equals 1 – the graphic stress state point will be on the strength surface, if the failure prediction number is less than 1 – the graphic stress state point will be under the strength surface, and if the scalar is more than 1 – the graphical stress state point will be above the strength surface. In the case of the 423 stress states, the failure prediction number was determined using eq. 2, 5, and 8 according to the von Mises, the Tsai-Wu, and the Ashkenazi strength criteria.

Results and discussion

The 423 stress states were transformed to the system of the anatomical main directions based on eq.11. 4 groups were created from the 423 stress states. 145 pieces were classified as $\sigma^{LL+}\sigma^{RR+}$ group, 103 pieces as $\sigma^{LL+}\sigma^{RR-}$ group, 113 pieces as $\sigma^{LL-}\sigma^{RR-}$ group, and 62 pieces as $\sigma^{LL-}\sigma^{RR+}$ group.

Calculated tensor components for the three strength criteria for each stress group are shown in table 2. The failure prediction numbers were determined in all cases of the four groups for the von Mises, the Tsai-Wu and the Ashkenazi strength criteria. The determined failure prediction numbers are shown in fig. 5 and table 3.

The failure prediction numbers calculated by the von Mises and the Tsai-Wu strength criteria in the $\sigma^{LL+}\sigma^{RR+}$ stress group are about 1. The value is 0.99 for the von Mises theory and 1.14 for the Tsai-Wu theory. The coefficient of variation is 72.1% for the von Mises theory and 97.6% for the Tsai-Wu theory, therefore the results are uncertain. In addition, negative failure prediction numbers were found in the failure prediction numbers determined by the von Mises and the Tsai-Wu theories. It means that the horizontal projection of the corresponding stress point is out of the projection of the strength surface. In that case the real failure prediction number is zero, because the stress point does not fit on the strength surface. The failure prediction numbers determined by the Ashkenazi strength criterion are around 1 in all stress groups and the coefficient of variation is equal to the variation of the strength of wood: n(I) = 0.87, n(II) = 0.75, n(III) = 0.88, n(IV) = 0.71, CoV(I) = 28.2%, CoV(II) = 24.4%, CoV(III) = 35.0%, and CoV(IV)=20.1%. The failure prediction numbers determined by the Ashkenazi strength criterion are less than 1. The spruce wood was weaker in the measurements performed by Szalai [2001] than in the experiments carried out by Eberhardsteiner [2002].

Table 2. Calculated tensor components for the von Mises, the Tsai-Wu, and the Ashkenazi strength criteria in the four stress groups

Tabela 2. Składowe tensorowe obliczone dla kryteriów wytrzymałości von Mises'a, Tsai-Wu i Ashkenazi'ego w czterech grupach naprężeń

S.crit. *Kryt. wyt.*	S.g. * *Gr. napr.* *	a_{LL} [MPa^{-1}]	a_{RR} [MPa^{-1}]	a_{LLLL} [MPa^{-1}]	a_{RRRR} [MPa^{-1}]	$a_{LLRR}+a_{RRLL}$ [MPa^{-1}]	$a_{LRLR}+a_{LRRL}+a_{RLLR}+a_{RLRL}$ [MPa^{-1}]
von Mises	I.	–	–	0.00025	0.02853	0.00748	0.01151
	II.	–	–	0.00025	0.08210	0.01985	0.01151
	III.	–	–	0.00041	0.08210	-0.04551	0.01151
	IV.	–	-	0.00041	0.02853	-0.01894	0.01151
Tsai- Wu	I.	-0.00452	-0.11761	0.00032	0.04840	0.01424	0.01151
	II.	-0.00452	-0.11761	0.00032	0.04840	0.01449	0.01151
	III.	-0.00452	-0.11761	0.00032	0.04840	-0.03862	0.01151
	IV.	-0.00452	-0.11761	0.00032	0.04840	-0.02391	0.01151
Ashkenazi	I.	–	–	0.01574	0.16892	0.14520	0.10730
	II.	–	–	0.01574	0.28653	0.05228	0.10730
	III.	–	–	0.02027	0.28653	0.02643	0.10730
	IV.	–	–	0.02027	0.16892	-0.02963	0.10730

* Stress groups are the following: I- $\sigma^{LL+}\sigma^{RR+}$; II- $\sigma^{LL+}\sigma^{RR-}$; III- $\sigma^{LL-}\sigma^{RR-}$; IV- $\sigma^{LL-}\sigma^{RR+}$

** Grupy naprężeń są następujące: I- $\sigma^{LL+}\sigma^{RR+}$; II- $\sigma^{LL+}\sigma^{RR-}$; III- $\sigma^{LL-}\sigma^{RR-}$; IV- $\sigma^{LL-}\sigma^{RR+}$*

Table 3. Failure prediction number values for the von Mises, the Tsai-Wu, and the Ashkenazi strength criteria

Tabela 3. Wartości wskaźnika przewidywania defektu dla kryteriów wytrzymałości von Mises'a, Tsai-Wu i Ashkenazi'ego

Strength criterion *Kryterium wytrzymałości*	Failure prediction numbers "n" [-] *Wskaźniki przewidywania defektu „n" [-]*			
	$\sigma^{LL+}\sigma^{RR+}$	$\sigma^{LL+}\sigma^{RR-}$	$\sigma^{LL-}\sigma^{RR-}$	$\sigma^{LL-}\sigma^{RR+}$
von Mises	0.99 (72.1)	0.27 (155.1)	0.48 (215.5)	1.29 (44.8)
Tsai-Wu	1.14 (97.6)	0.78 (100.0)	0.47 (165.5)	1.38 (51.5)
Ashkenazi	0.87 (28.2)	0.75 (24.4)	0.88 (35.0)	0.71 (20.1)

Coefficient of variation [%] is given in parentheses.

Współczynnik zmienności [%] podano w nawiasach.

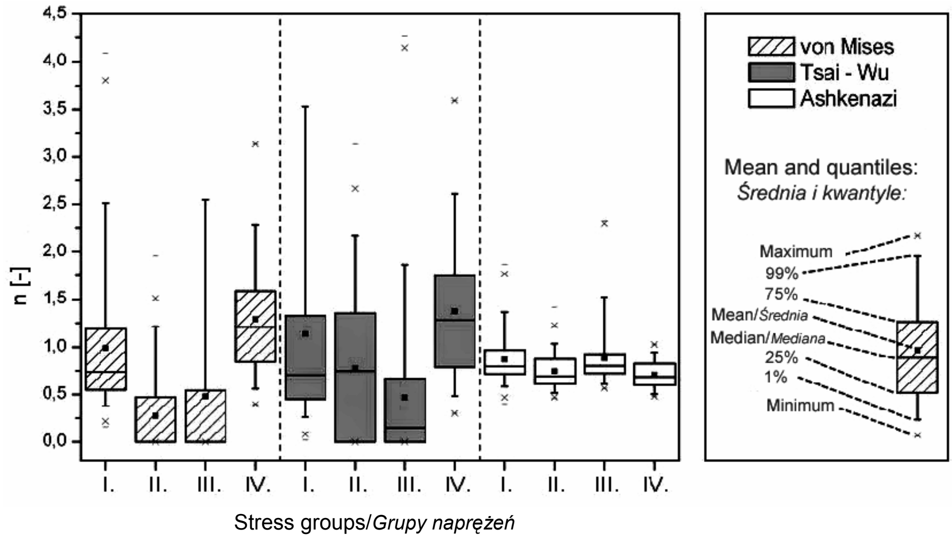

Fig. 5. Failure prediction number values corresponding to different criteria in the four stress groups
Stress groups are the following: I- $\sigma^{LL+}\sigma^{RR+}$ **; II-** $\sigma^{LL+}\sigma^{RR-}$ **; III-** $\sigma^{LL-}\sigma^{RR-}$ **; IV-** $\sigma^{LL-}\sigma^{RR+}$

Rys. 5. Wartości wskaźnika przewidywania defektu odpowiadające różnym kryteriom w czterech grupach naprężeń
Grupy naprężeń są następujące: I- $\sigma^{LL+}\sigma^{RR+}$ ***; II-*** $\sigma^{LL+}\sigma^{RR-}$ ***; III-*** $\sigma^{LL-}\sigma^{RR-}$ ***; IV-*** $\sigma^{LL-}\sigma^{RR+}$

The results are reliable based on the following observations:

- for all of the three theories tensor components were calculated from the same engineering strength, therefore the differences among the theories could be marked,
- based on the investigation of the variance values coefficients of variation of the Ashkenazi theory represent the variance values of the mechanical properties of wood; the variance values of the von Mises and the Tsai-Wu theories are extreme,
- although the failure prediction numbers (n) are not exactly 1 in the case of the Ashkenazi strength criterion, calculated values were around 1 (or slightly below) in all four groups – a better mechanical property of the sample material tested in Vienna is considered to be the reason for that.

Conclusions

The present study demonstrates that the comparison of anisotropic strength criteria is useful. This research compared three anisotropic strength criteria (the von Mises criterion, the Tsai-Wu criterion, and the Ashkenazi criterion) based on experimental data for spruce wood. Large number of tested specimens (over 400) was checked in terms of the strength criteria, therefore the results are acceptable.

After calculating the failure prediction numbers for the three theories, it was found that the means and the variance values are acceptable only for the Ashkenazi strength criterion, because the mean values can predict the failure and the variance is similar to the variance of wood properties.

After theoretical approach [Szalai 2008], experimental results confirm that in the planar stress state the Ashkenazi strength criterion best describes the failure of wood.

In future investigations the following topics are of interest to us:
- selection of the ideal triaxial test method,
- comparison of anisotropic strength criteria in triaxial stress states,
- investigation of other species in multiaxial stress state.

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PORÓWNANIE ANIZOTROPOWYCH KRYTERIÓW WYTRZYMAŁOŚCI W DWUOSIOWYM STANIE NAPRĘŻEŃ

Streszczenie

Ze względu na skomplikowaną mikrostrukturę, właściwości fizyczne i mechaniczne drewna i materiałów drewnopochodnych są anizotropowe. Z powodu tej złożonej struktury, przewidywanie wytrzymałości drewna jest trudne i komplikuje się jeszcze bardziej w złożonych (dwu- i trójosiowych) stanach naprężeń.

W niniejszej pracy porównano trzy fenomenologiczne anizotropowe kryteria wytrzymałości (von Mises'a, Tsai - Wu i Ashkenazi'ego) na podstawie danych doświadczalnych uzyskanych uprzednio dla świerka (*Picea abies* [L.] Karst.). Celem tego porównania było ustalenie, która teoria wytrzymałościowa przewiduje właściwie powstawanie defektów w drewnie w dwuosiowym stanie naprężeń.

Na podstawie posiadanych danych doświadczalnych wyznaczono wartości wskaźników przewidywania defektu wraz z ich współczynnikami zmienności.

Uzyskane wyniki pokazują, że w płaskim stanie naprężeń kryterium wytrzymałościowe Ashkenazi'ego najlepiej przewiduje powstawanie uszkodzeń w drewnie.

Kolejnym etapem analiz będzie porównanie anizotropowych kryteriów wytrzymałościowych dla trójosiowego stanu naprężeń oraz badania innych gatunków drewna w wieloosiowych stanach naprężeń.

Słowa kluczowe: anizotropowe kryteria wytrzymałości, badania dwuosiowe, płaskie stany naprężeń, wskaźnik przewidywania defektu, kryterium wytrzymałości Ashkenazi'ego