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THE ROLE OF SHEAR STRESS IN THE BENDING STRENGTH TEST OF SHORT AND MEDIUM LENGTH SPECIMENS OF CLEAR WOOD

The process of wood bending displays measurable departures from the standard beam theory. The most well-known departures are caused by shear. In the mathematical part of this work we found and analysed full plane stress of wood which occur in III-point and IV-point loading. Hill-type strength criteria have been used. The role of shear is determined by the relation between shear span-depth ratio and bending-shear strength ratio. Three types of this relation have been defined, one of which specifies the concept of medium length beam. In the second part of this work we statistically described bending-shear strength ratio for European, American and exotic wood species. On this basis, we determined the shear span-depth ratio of the medium beam. The role of shear stress in the bending strength test of medium beam cannot be omitted. The work contains elements of strength theory from a historical perspective, especially concerning strength criteria. Given one result of the contact mechanics.

Keywords: bending-shear strength ratio, shear span-depth ratio, bending with shear, Zhuravskii formula, Hertz pressure, Hill strength criterion, Norris criterion, short-medium-long beam, length conditions, log-normal distribution

Introduction

The principles of the beam theory were introduced circa 1750 by two Swiss scholars, Leonard Euler and Daniel Bernoulli [Timoshenko 1953].

Let us consider a straight wooden beam positioned horizontally, along the x -axis. Let us assume that the load forces of the beam and the reactive forces of the supports, act vertically, i.e. parallel to the y -axis (fig. 1).

It is assumed that the normal strains and stresses in the cross-section of the beam (near top support) are proportional to the distance from the neutral axis (fig. 2a):

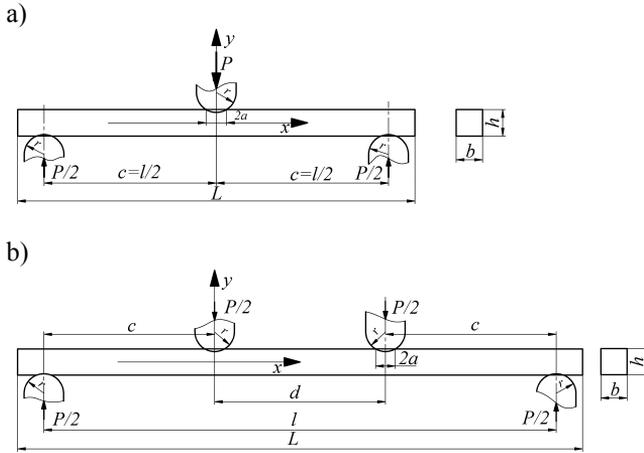


Fig. 1. Diagram of bending: a) III-point, b) IV-point

$$\sigma_{\parallel}(y) = -\sigma_m \cdot 2y/h, \quad \sigma_m = 3Pc/(bh^2) = R \quad (1a, 1b)$$

where: $\sigma_{\parallel} = \sigma_{xx}$ – normal stress parallel to grain, y – coordinate, σ_m – maximum stress (equal R for maximum P), h – beam depth, P – resultant beam loading (maximum) force, c – distance from the load point to the nearest point of bottom support (half shear span), b – beam width, R – bending strength. The formula (1b) applies to both methods, the III-point and the IV-point, in accordance with old standards for small samples [BS 373:1957]¹ or [PN-68/D-04103] equivalent to [PC 022-67]. The newer standards for small samples are used for the III-point method [e. g. ISO 13061] and for structural-size timber IV-point method [e. g. ISO 8375].

The distribution of shear stresses in a beam cross-section can be determined using Zhuravskii² formula [Zhuravskii 1856]. For a rectangular beam, it leads to the following parabolic distribution of shear stress (fig. 2b):

$$\tau(y) = -\tau_m [1 - (2y/h)^2], \quad \tau_m = 3P/(4bh) = 1.5\tau_0 \quad (2a, 2b)$$

where: $\tau = \sigma_{xy}$ – shear stress; τ_m , τ_0 – maximum and mean absolute values of shear stress.

According to Hertz theory [Hertz 1881], contact stress (fig. 2c) is equal to:

$$\sigma_{\perp}(x) = -\sigma_c \sqrt{1 - (x/a)^2}, \quad \sigma_c = 4P/(\pi bw) = (4/\pi) \cdot \sigma_0 \quad (3a, 3b)$$

where: $\sigma_{\perp} = \sigma_{yy}$ – compression stress perpendicular to grain, x – coordinate; σ_c , σ_0 – maximum and mean absolute values of stress, a – half width contact of support, w – total width contact of top supports (equal $2a$ or $4a$). The value of

¹Where the IV-point method is a more accurate determination of MOE.

²D.J. Zhuravskii was a renown Russian engineer.

a is an experimental variable, but we can try to estimate it. Contact mechanics handbook [Johnson 1985] gives a for cylinder and sphere press on half-space:

$$a = \sqrt{4 Fr / (\pi b E_{\perp})}, \quad a = \sqrt[3]{3 Fr / (4 E_{\perp})} \quad (4a, 4b)$$

where: F – loading force (equal P or $P/2$), r – radius of cylindrical or spherical support, E_{\perp} – modulus of elasticity perpendicular to grain.

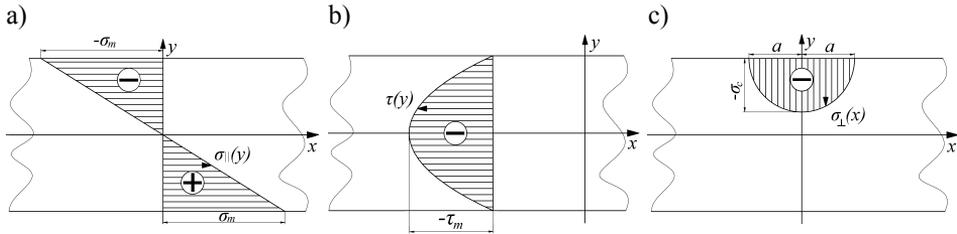


Fig. 2. Stress distributions in the bending beam of homogeneous material: a) normal parallel to grain, b) shear, c) compression of the support

Full plane stress should satisfy the differential equations [e.g. Johnson 1985]:

$$(\sigma_{\parallel})'_x = -\tau'_y, \quad (\sigma_{\perp})'_y = -\tau'_x \quad (5a, 5b)$$

Complex distribution of plane stress requires application of proper strength criteria. There are numerous criteria referring to the strength of a material, two of which refer directly to shear. It turns out that shear stresses occur in a material even during pure tension or pure compression³. The measure of material effort in the Coulomb-Tresca criterion is double the maximum value of the shear stress [Tresca 1864]. Huber criterion [Huber 1904] stipulates that the measure of material’s effort in a complex state is such a value of normal stress which gives the same distortion strain energy as the stress state. Therefore:

$$\sigma_{red} = \sqrt{\sigma_{\parallel}^2 - n \sigma_{\parallel} \sigma_{\perp} + \sigma_{\perp}^2 + m \tau^2} \leq R_{red} \quad (6a/b)$$

where: σ_{red} – reduced stress; $n = 2, m = 4$ in the Coulomb-Tresca criterion (6a); $n = 1, m = 3$ in the Huber criterion (6b); R_{red} – effective strength.

The first to identify the distortion strain (shear) energy and to publish the criterion in form (6b) was a renowned Polish engineer Maximilian Huber⁴.

The character of both criteria makes them applicable to elastic-plastic isotropic materials, but wood is anisotropic. For example, formula (6) implies that shear strength is only \sqrt{m} (2 or 1.7) times smaller than the effective

³This fact could be described by the Mohr’s circle. However, in experimental tension tests for metals, there occur diagonal 45° fractures, called Chernov-Lüders lines.

⁴It was formed independently by von Mises in 1913 and Hencky in 1924. The same idea was proposed by Maxwell in 1856, published in 1936 [e.g. Kordzikowski 2012].

strength (bending); however, for wood is it about 10 times smaller [Kollmann and Côté 1984]. Therefore, a simple inequality, called maximum stress criterion⁵ [e.g. Camanho 2002], is often used in wood technology and composite materials:

$$|\sigma_{\parallel}| \leq R_{\parallel}, \quad |\sigma_{\perp}| \leq R_{\perp}, \quad |\tau| \leq S \quad (7)$$

where: R_{\parallel} , R_{\perp} – strength along and across the fibres, S – shear strength (along the fibres). However, more complicated criteria for anisotropic materials should be applied in a complex stress. The first such criterion was introduced by von Mises [von Mises 1928]. His criterion is discussed in the presentation [Zahr Vinuela and Perez Castellanos 2015]. The Mises criterion was simplified by Hill [Hill 1948] in such a way that it would be reduced to Huber criterion in case of an isotropic material⁶. For monotropic materials⁷ in plane stress parallel to the monotropic axis, this criterion has the following form:

$$H = \sigma_{\parallel}^2 / R_{\parallel}^2 - \sigma_{\parallel} \sigma_{\perp} / (R_{\parallel} R_*) + \sigma_{\perp}^2 / R_{\perp}^2 + \tau^2 / S^2 \leq 1 \quad (8a/b)$$

where: H – measure of material effort, $R_* = R_{\parallel}$ in Hill criterion (8a), $R_* = R_{\perp}$ in Norris criterion (8b).

Hill described the case of rotational symmetry of anisotropy and plane stress perpendicular to the axis of symmetry. The case of parallel plane was described 18 years later [Azzi and Tsai 1965]. Therefore, formula (8a) is sometimes called Azzi-Tsai criterion [e.g. Guindos 2014] (or Tsai-Hill criterion [e.g. Kolios and Proia 2012]), and its general form is the Tsai-Hill criterion [e.g. Camanho 2002]. Three years earlier, a similar criterion (8b) had been proposed by Norris [Norris 1962].

None of these criteria takes into consideration differences in compression and tension strength⁸. The problem was solved by Hoffman [Hoffman 1967] or Tsai and Wu [Tsai and Wu 1971]. A more complicated solution had already been presented in 1966 [Gołdenblat and Kopnov 1966]⁹.

Methods and data sources

The first theoretic objective of this work is a mathematical analysis of the effect of shear on the bending strength of wood. At the start we need to find full plane stress satisfying (5). The purpose was to obtain the function of wood effort in a cross-section which would take shear into account. This function was tested for cases of different quality. Single-parameter strength criteria (6) do not offer the correct ratio R/S of bending and shear strength for wood (see this *ratio* definition

⁵Not to be confused with Rankine-Clebsch criterion of maximal normal stress.

⁶Hill's criterion contains 6 parameters, and Mises as many as 9.

⁷Wood is approximately transversely isotropic (monotropic).

⁸In a nonanalytical approach, separated signs R_{\parallel}^{\pm} , R_{\perp}^{\pm} [e.g. Garab and Szalai 2010].

⁹According to Kyzioł [2009], it is practically equivalent to Tsai-Wu's criterion.

(20) in the next section). Research tools in this work are Hill¹⁰ and Norris strength criteria in the form of (8) for monotropic materials such as wood. Because of the moving of the neutral axis during wood bending, it is pointless to apply more complicated criteria such as Hoffman criterion or Tsai-Wu criterion. The specific theoretical purpose of this work is to form general conditions for a beam under bending where the effect of shear on strength can be ignored.

The second, practical part of the work, presents a study of the relation of bending (III-point method) and shear strengths based on three comprehensive data sources for domestic [Krzysik 1978], American [Green et al. 1999] and exotic wood species [Jankowska et al. 2012]. The last source contains results of a study carried out by one of the co-authors of this work. All three data sources include the results of tests of small clear wood specimens related to moisture content of 12% or Krzysik cases to 15%¹¹. American data sources are consistent with [ASTM Standard D 143-94]. Shear was measured by compression L-shaped specimens to failure on 2×2 inches tangential (or radial) surface. In (tangential) bending tests, $2 \times 2 \times 30$ (or $1 \times 1 \times 16$) inch specimens were used, with a span length of 28 (or 14) inches and a radius of centre support of 3 (or 1.5) inches. The (tangential) bending strength of Polish and exotic species was determined by standard [PN-77/D-04103], which complies with [ISO 3133], but is not equivalent to it. The specimens had a size of $20 \times 20 \times 300$ mm with a span length of 240 mm and radius of the supports of 15 mm. The shear strength in Krzysik data sources was measured in accordance with the old standard [PN-59/D-04105], but for exotic species a newer standard [PN-79/D-04105] complies with [ISO 3347]¹². In the old standard, compression L-shaped specimens were sheared on a 20×20 mm radial surface. In the second standard, T-shaped specimens with radial or tangential shear surface size of 20×30 mm were used. The source data were statistically analysed. For log-normal distribution [Gaddum 1945] the following designations were used:

$$X = M_{-\delta}^{+\Delta} = e^{\mu \pm \sigma}, \quad X_m = e^{\mu + 3\sigma} \quad (9a, 9b)$$

where: X – log-normal variable, M – median value; Δ , δ – right and left deviation, μ – mean value of $\ln X$; σ – standard deviation of $\ln X$; X_m – statistic extreme of X (alpha level 0.001). Pearson correlation coefficients ρ_i were calculated for each source between bending strength and shear strength or ρ_i' between their logarithms. We compared them on alpha level 0.001 with the critical values $\rho_i^{\text{crit}}(n_i)$ for the variables uncorrelated with n_i data points¹³:

$$\rho_1^{\text{crit}}(29) = 0.58, \quad \rho_2^{\text{crit}}(206) = 0.23, \quad \rho_3^{\text{crit}}(40) = 0.50 \quad (10)$$

¹⁰R. Hill was an English applied mathematician. He passed away in 2011.

¹¹For *ratio* parameter it is practically irrelevant.

¹²About Polish standard of shear strength: Kozakiewicz [2000].

¹³Alpha value 0.001 is much more restrictive here than 0.05.

Computed measure compliance λ_i of *ratio* distributions with normal distribution and λ'_i with log-normal distribution. The λ -Kolmogorov test was used with the Lilliefors corrections [Lilliefors 1967] and its newer tables [Molin and Abdi 1998]. Cross compliance of *ratio* distributions tested by measure λ_{i-j} of Kolmogorov-Smirnov. The tests were performed on alpha level 0.05 (or 0.01), which implies the critical values of λ statistic:

$$\lambda_i^{\text{crit}}=0.88, \quad \lambda_{i-j}^{\text{crit}}=1.36, \quad (\lambda_{i-j}^{\text{crit}}=1.63 \quad \text{for } \alpha=0.01) \quad (11)$$

The average *ratio* was determined and extreme cases were analysed. On the basis of these values was derived numeral length conditions for beam, which the role of shear stress in the bending strength test should not be omitted. These conditions are expressed by the key shear span-depth ratio $2c/h$.

Mathematical research

Consider the plane stress during the bending test (fig. 1 and 2). The greatest stresses occur in the area under the top support $|x| \leq a$, $|y| \leq h/2$. Full plane stress (fig. 3) is found by solving the equations (5) in compliance with (3), (2), (1)¹⁴:

$$\sigma_{\parallel}(x, y) = -\sigma_m \frac{2y}{h} \left\{ 1 - \frac{4a}{\pi l} \left[\frac{x}{a} \arcsin(x/a) + \sqrt{1-(x/a)^2} - \frac{1}{3} \sqrt{1-(x/a)^2}^3 \right] \right\} \quad (12)$$

$$\tau(x, y) = \tau_m \left[1 - (2y/h)^2 \right] \cdot \frac{2}{\pi} \left[\arcsin(x/a) + \frac{x}{a} \sqrt{1-(x/a)^2} \right] \quad (13)$$

$$\sigma_{\perp}(x, y) = -\sigma_c \sqrt{1-(x/a)^2} \cdot (1/4) \cdot [2 + 3(2y/h) - (2y/h)^3] \quad (14)$$

Equations (12), (13) apply to III-point method, but figure 3 shows graphs of stresses for both III-point and IV-point methods.

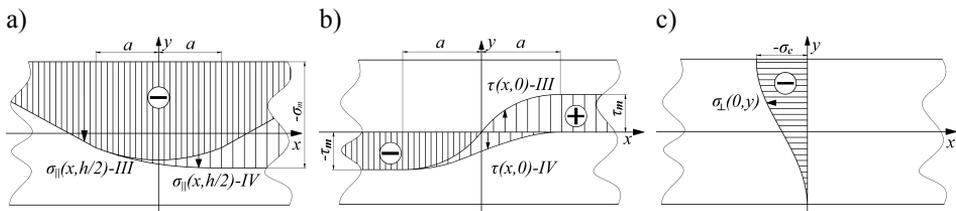


Fig. 3. Stress distributions under the loading support: a) normal parallel to grain, b) shear, c) compression of the support

Figure 3 shows the dependence of one variable, and the other is the same as in figure 2. According to criteria (8a/b), stress σ_{\perp} doesn't affect the strength if:

¹⁴These are the boundary conditions if we take into account arm's factor $(c-a)/c$.

$$-\frac{\sigma_{\parallel}\sigma_{\perp}}{R_{\parallel}R_*} + \frac{\sigma_{\perp}^2}{R_{\perp}^2} \leq 0 \quad \xrightarrow{x=0, y=h/2} \quad \frac{2c}{h} \geq \frac{2}{3} \cdot \frac{4}{\pi} \cdot \frac{h}{w} \cdot \left(\frac{R_{\parallel}R_*}{R_{\perp}^2} + \frac{2a^2}{h^2} \right) \quad (15a/b, 16a/b)$$

The inequality (16a/b) was derived from (12), (1b), (14), (3b) and generalization of (12) to IV-point method. Let $h/w = 1$ ([DIN 52186] or [EN 408])¹⁵ and $R_{\parallel}/R_{\perp} \approx 5.2$ for compression (acc. [Krzysik 1978]). Then we get $2c/h > 23.0$ from (16a) ($R_* = R_{\parallel}$) and $2c/h > 4.4$ from (16b) ($R_* = R_{\perp}$). Here the Hill criterion is too strong and isn't compatible with the experiment, but we have a weaker condition (16b) resulting from Norris criterion¹⁶.

Assuming (16b) and (14) in the analysis of the role of shear, we can put $\sigma_{\perp} = 0$ for $|x| = a$. Now, Hill and Norris criteria (8a/b) take the following form:

$$H(y) = (\sigma_{m(x)}/R)^2 \cdot (2y/h)^2 + (\tau_m/S)^2 \cdot [1 - (2y/h)^2]^2 \leq 1 \quad (17)$$

where: $\sigma_{m(x)}$ – maximum value of σ_{\parallel} for a fixed x ; $R = R_{\parallel}$ – bending strength, $h/2 \leq y \leq h/2$. If we derive the ancillary variable $t = 1 - (2y/h)^2$, the condition (17) will be reduced to a quadratic inequality:

$$H(t) = (\sigma_{m(x)}/R)^2 - (\sigma_{m(x)}/R)^2 \cdot t + (\tau_m/S)^2 \cdot t^2 \leq 1, \quad 0 \leq t \leq 1 \quad (18)$$

The variable t for the middle layer of the beam has the value of 1, and 0 for the outer layers. The argument of vertex (minimum) of this square function is:

$$t_m = (S/R)^2 \cdot \sigma_{m(x)}^2 / (2\tau_m^2) \quad (19)$$

The formula contains the key *ratio* of bending and shear strengths:

$$ratio = R/S \quad (20)$$

If $t_m \geq 1$, then effort H has the lowest value in the middle layer of the beam ($t = 1$) and increases monotonically towards the outer layers ($t = 0$). The maximum normal stress is significantly greater than the greatest shear stress (at least 1.41 *ratio* times):

$$\sigma_{m(x)}/R \geq \sqrt{2} \cdot \tau_m/S \quad \text{– normal bending} \quad (21a)$$

If $0.5 < t_m < 1$, then local maximum in the middle layer of the beam ($t = 1$) is not the global (fig. 4b). The shear stresses are not dominating, but should not be overlooked:

$$\tau_m/S < \sigma_{m(x)}/R < \sqrt{2} \cdot \tau_m/S \quad \text{– bending with shear} \quad (21b)$$

Local maximum in the middle layer of the beam ($t=1$) is also the global maximum $t_m \leq 0.5$. It is a situation of domination of the shear stresses:

$$\sigma_{m(x)}/R \leq \tau_m/S \quad \text{– shear bending} \quad (21c)$$

¹⁵ These standards recommend the use steel plates under supports.

¹⁶ The easiest derivation of this condition is from criterion (7).

Results of mathematical research and discussion

Inequalities (21) lead to the following shear span length conditions of a beam¹⁷:

– long beam $2c/h \geq ratio/\sqrt{2} + 2a/h$ (22a)

– medium beam $ratio/2 + (16a - 4w)/(3\pi h) < 2c/h < ratio/\sqrt{2} + 2a/h$ (22b)

– short beam $ratio/2 + (16a - 4w)/(3\pi h) \geq 2c/h$ (22c)

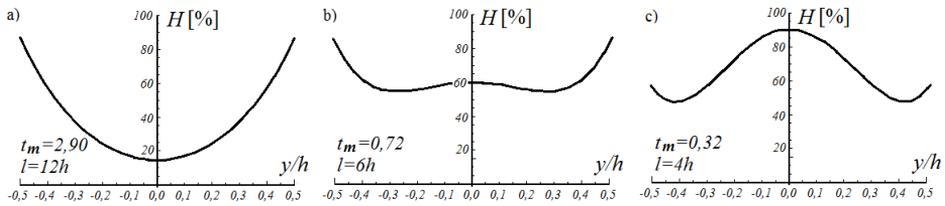


Fig. 4. Graphs of wood effort $H(y)$ in cross-section ($ratio=10$): a) long beam, b) medium beam, c) short beam

Conditions (22a) or (22b) and (16b) should be met if we intend to test bending, not shear, in the bending test. For example, in the conference materials [Sorn et al. 2011], bending strength of spruce is 95% for $2c/h \approx 12$. From (22a) we conclude that here maximal $ratio \approx 17$.

Conditions (22c) and (16b) should be met if we wish to measure the shear strength using the short-beam test [BS EN ISO 14130]. This method can also be used to try for wood [Yoshihara and Furushima 2003]. Standard [ASTM D 198-02] recommends here $2c/h < 10$ for IV-point bending method. Flexural fracture curves described in [Schneeweiß and Felber 2013] have shear fracture for $2c/h \leq 8$. This suggests, under (22c), that the maximal value of $ratio$ is approximately 16.

On the basis of (14), we calculated half width of support-beam contact:

$$a = \sqrt[3]{2Frhl/(\pi bE_{\perp})} \quad (23)$$

whose values are approximately 150%, 95% of the (4a), (4b) respectively. Formula (23) shows that the radius of top supports in IV-point method would be four times smaller than in III-point method.

Data sources research

Dimensionless $ratio$ parameter expressed with formula (20) determines the susceptibility of wood to shear during bending. Tables 1, 2 and 3 present the extreme values of this parameter for various wood species from three different sources.

¹⁷ For both III-point and IV-point methods, like (16).

Table 1 presents selected data from the most recognised Polish sources. The correlation coefficient for all these 29 species is equal to $\rho_1 = 0.612 > 0.58$ or $\rho_1' = 0.77 > 0.58$. In figure 5.1 we see a histogram of these ratio values.

Table 1. Bending and shear strength of Polish market wood [Krzysik 1978]

The trade and Latin name of wood according to [EN 13556:2003]	Bending strength R [MPa] ¹⁸	Shear strength S [MPa] ¹⁸	Ratio (<i>ratio</i>) R/S
European lime (<i>Tilia cordata</i> Mill.)	90	4.5	20.0
Common alder (<i>Alnus glutinosa</i> (L.) Gaerthn.)	85	4.5	18.9
European walut (<i>Juglans regia</i> L.)	119	7.0	17.0
European beech (<i>Fagus sylvatica</i> L.)	105	8.0	13.1
European ash (<i>Fraxinus excelsior</i> L.)	102	12.8	8.0
Robinia (<i>Robinia pseudoacacia</i> L.)	120	16.0	7.5
European aspen (<i>Populus tremula</i> L.)	52	7.0	7.4
Yew (<i>Taxus baccata</i> L.)	88	14.0	6.3
Mulberry* (<i>Morus alba</i> L.)	73	12.5	5.8

*The wood database <http://www.wood-database.com>.

The Kolmogorov statistic gives parameter $\lambda_1 = 0.87 \approx 0.88$ or $\lambda_1' = 0.54 < 0.88$, which means log-normal distribution (and even normal) on alpha level 0.05. Let us take a look at species less susceptible to shear during bending. Here are the most ring-porous species: mulberry with minimal *ratio* 5.8, robinia – 7.5 and ash – 8.0. Also, softwood yew has a very low value – 6.3.

Bending strength tests should be performed for the tree species most susceptible to shear. Of the species indigenous to Poland, there are three which stand out: European lime – 20.0, black alder – 18.9 and European walnut – 17.0. These are diffuse-porous or semi-ring-porous species. Quite a high *ratio* value shown by European lime can be explained by its relatively high bending strength, for such light wood, and by its large medullary rays, which lower the shear strength. European lime is a perfect material for sculpting, not only thanks to its softness but also thanks to its low shear strength.

Examples of species from the USA market with extreme disproportion are presented in table 2 below. The correlation coefficient for 206 species is $\rho_2 = 0.813 > 0.23$ or $\rho_2' = 0.86 > 0.23$, which is a strong correlation. The histogram in fig. 5.2) presented the *ratio* of all the species for this source. Kolmogorov statistic¹⁹ gives values $\lambda_2 = 0.94 > 0.88$ and $\lambda_2' = 0.46 < 0.88$, which mean log-normal distribution, but not normal. It is caused by three species: marishballi, mersawa, kaneelhart, whose *ratio* is higher than a triple standard deviation.

¹⁸ Original unit is 0.1 kG/cm² = 0.981 MPa, but for 15% moisture content (not 12%).

¹⁹ For full experimental distribution function.

Table 2. Bending and shear strength of USA market wood [Green et al. 1999]

The trade and Latin name of wood according to [EN 13556:2003]	Bending strength <i>R</i> [MPa]	Shear strength <i>S</i> [MPa]	Ratio (<i>ratio</i>) <i>R/S</i>
Marishballi* (<i>Licania sparsipilis</i> S. F. Blake)	191	12.1	15.8
Mersawa (<i>Anisoptera spp.</i>)	95	6.1	15.6
Kaneelhart* (<i>Cinnamomum camphora</i> Ness & Eberm)	206	13.6	15.2
Manbarklak* (<i>Eschweilera longipes</i> Miers.)	183	14.3	12.8
Western hemlock (<i>Tsuga heterophylla</i> (Raf.) Sarg.)	81	6.5	12.5
Ipé (<i>Tabebuia spp.</i>)	175	14.2	12.3
‘White cedar’ (<i>Thuja occidentalis</i> L.)	42	6.9	6.1
Silver maple** (<i>Acer saccharinum</i> L.)	61	10.2	6.0
American white oak (<i>Quercus alba</i> L.)	71	12.5	5.7
Peroba rosa (<i>Aspidosperma peroba</i> Fr. Allem.)	83	17.2	4.8

*[Green et al. 1999].

**The wood database (<http://www.wood-database.com>).

Other species most susceptible to shear during bending are manbarklak – 12.8 and western hemlock – 12.5. The less susceptible to shear during bending are white oak – 5.7 and silver maple – 6.0.

The *ratio* parameter values for exotic species is presented in last table 3.

Table 3. Bending and shear strength for exotic wood [Jankowska et al. 2012]

The trade and Latin name of wood according to EN 13556:2003	Bending strength <i>R</i> [MPa]	Shear strength <i>S</i> [MPa]	Ratio (<i>ratio</i>) <i>R/S</i>
Obeche (<i>Triplochiton scleroxylon</i> K.Schum.)	73	4.0	18.2
Azobe (<i>Lophira alata</i> Benks. ex Gaertn.f.)	246	15.0	16.4
African padouk (<i>Pterocarpus soyauxii</i> Toub.)	134	8.5	15.8
Tatajuba (<i>Bagassa guianensis</i> Aubl.)	109	7.0	15.6
Pterygota (<i>Pterygota macrocarpa</i> K.Schum.)	86	7.0	12.3
Gombeira (<i>Melanoxylon brauna</i> Schott.)	182	23.9	7.6
Pau amarelo (<i>Euxylophora paraensis</i> Huber.)	125	16.6	7.5
Bintangor (<i>Calophyllum spp.</i> , e.g. <i>C. inophyllum</i> L.)	80	11.5	7.0
Kempas (<i>Koompasia malaccensis</i> Maing.ex Benth.)	110	16.0	6.9
Avodire (<i>Turraenthus africanus</i> (Welw.ex C.DC.) Pellegri.)	86	12.5	6.9

For all 40 species $\rho_3 = 0.728 > 0.50$ or $\rho_3' = 0.76 > 0.50$ and $\lambda_3 = 1.12 > 0.88$ or $\lambda_3' = 0.73 < 0.88$, which means good correlation and log-normal distribution. On the histogram in figure 5.3, we see that four species stand out from the rest. These are obeche, azobe, tatajuba and African padouk.

The lowest susceptibility to shear is shown by avodire wood, with *ratio* 6.9, consistent with the American source where *ratio* is 6.3. For kempas wood, we do not see this compatibility. An exotic species more susceptible to shear during bending is, for instance, pterygota – 12.3.

Let us now use the Kolmogorov-Smirnov test to study the compliance of the *ratio* distributions with the three sources. Such compliance of the data occurs only for the exotic and Polish wood species $\lambda_{1-3} = 0.58 < 1.36$. However, Polish and American sources show a discrepancy $\lambda_{1-2} = 1.80 > 1.36$, similar to the data for exotic and American species $\lambda_{2-3} = 1.61 > 1.36$. But the last two distributions show compliance on alpha level 0.01 ($\lambda_{2-3} = 1.61 < 1.63$).

Results of data research

The *ratio* showed log-normal distribution at a significance level of 0.05 (or more) in all three sources (fig. 5). These distributions show mutual conformity at a significance level of 0.01, except for the American and the older Polish sources.

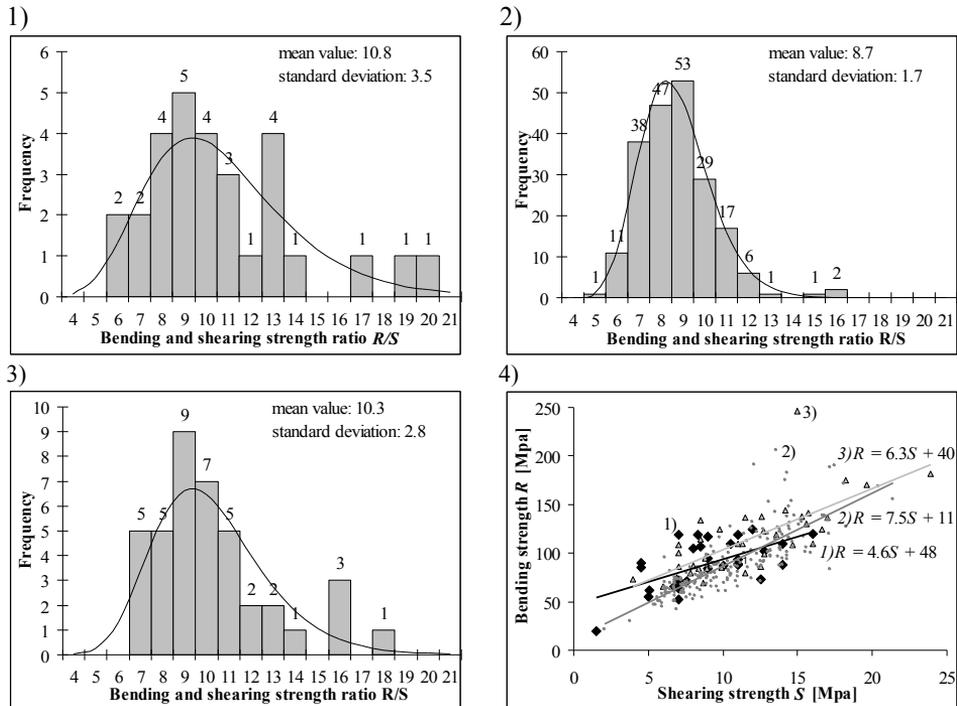


Fig. 5. Histograms (1-3) of the bending and shear strength *ratio* acc. : 1) Krzysik [1978], 2) Green et al. [1999], 3) Jankowska et al. [2012], 4) all sources – correlation

All histograms have the same dominant value of 9. The medians of *ratio* values, together with the right and left deviations, for all sources are:

$$ratio_1 = 10.3_{-2.7}^{+3.6}, \quad ratio_2 = 8.6_{-1.4}^{+1.7}, \quad ratio_3 = 9.9_{-2.2}^{+2.8} \quad (24)$$

where the arithmetic mean is 9.6. Statistic extremes of *ratio* are 25.4, 14.9 and 20.9 respectively (the arithmetic mean is 20.4). American data 2) have the best correlation and the smallest deviations. Exotic species 3) *ratio* distribution parameters have intermediate values. On the basis of the conformity of this source with sources 1) and 2), it can be assumed that the $ratio = 9.9_{-2.2}^{+2.8}$ is the value for wood species from all over the world. For comparison, other authors assume that the “lower limit” of *ratio* is 10 [Soltis and Rammer 1997]²⁰.

Conclusions

The analysis of the bending strength test of a beam that was not enough long showed that the role of shear may be higher than the role of the bending moment and support crushing. For crushing stresses, Norris criterion was better than Hill criterion (according to Rowlands et al. [1985]). Three types of bendings were described as: normal bending (for a long beam), bending with shear (for a medium beam) and shear bending (for a short beam). The introduction of an intermediate state for a medium beam is a novelty. In this state of bending, shear stresses dominate locally, but not globally. In other words, the wood effort of the global minimum changes to the local maximum.

During the bending strength test, the shear span-depth ratio $2c/h$ for a medium beam should be greater than half of the bending-shear *ratio* R/S – (22b). For a long beam, shear span-depth ratio should be greater than bending-shear *ratio* divided by root of two – (22a).

On the basis of (22b), (24) the shear span of the medium beam can be determined for²¹:

$$\text{– Polish source:} \quad 5.1_{-1.3}^{+1.8} < 2c/h < 7.3_{-1.9}^{+2.6} \quad (25.1)$$

$$\text{– American source:} \quad 4.28_{-0.72}^{+0.86} < 2c/h < 6.1_{-1.0}^{+1.2} \quad (25.2)$$

$$\text{– exotic wood:} \quad 5.0_{-1.1}^{+1.4} < 2c/h < 7.0_{-1.5}^{+2.0} \quad (25.3)$$

Statistic extremes of medium beam shear span depth ratio are 18.0, 10.5 and 14.7 respectively (the arithmetic mean is 14.4). By analogy, the maximal shear span depth ratio for a short beam is 12.7, 7.4, 10.4 respectively (mean 10.2). For comparison, the standards [BS 373:1957, PN-68/D-04103, ISO 3133, DIN 52186, ASTM D 198-02, EN 408] give the shear span depth ratio $2c/h$ equal: 14 (or 6^{iv}), 12 or 8^{iv} , 14 ± 2 , 15, 10^{iv} - 24^{iv} , $12^{iv} \pm 2$, respectively²². It determines the

²⁰ However, their diagrams in figures 2, 3, 4, 5 suggest an upper limit.

²¹ We disregard here the width contact (w and a).

²² Superscript iv refers to the IV-point bending method.

correct shear span length of long beams but doesn't exclude the extremes of medium beams. The exception is the old standard [PN-68/D-04103] (or [BS 373:1957]), which suggests too short beam in the IV-point bending method.

One should be careful with the species more susceptible to shear, i.e. European lime, common alder, European walnut or western hemlock. Thanks to tables like 1-3, we can match the span to a given wood species. For the species that are less susceptible to shear during bending, such as mulberry, yew, robinia or American white oak, silver maple and avodire, even 30% shorter beams could be used. Here, caution should be exercised that the specimen is not crushed (condition (16b)).

In the case of IV-point bending, the values of shear span should practically equal that of III-point bending. However, the IV-point method is less prone to wood crushing by loading support. Moreover, only this method has exact stress σ_m given by (1b), without other stress. Interestingly, this fact doesn't depend on the size of the supports.

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