Rifat KURT, Selman KARAYILMAZLAR

WHICH CONTROL CHART IS THE BEST FOR THE PARTICLEBOARD INDUSTRY: SHEWHART, CUSUM OR EWMA?

In this study, control charts were prepared using data received from a particleboard manufacturer to determine the factors that impair quality, and the most suitable control charts for the company were specified accordingly. The study material consisted of the tensile strength values of boards taken from a factory over three months. Shewhart, CUSUM and EWMA control charts were used for variable quantities to achieve the targeted quality level. It was concluded from the research that interdependent and independent evaluation of the data observed during particleboard manufacturing was required; therefore, the combined use of CUSUM and Shewhart control charts was proposed.

Keywords: Statistical Quality Control, particleboard industry, Shewhart, CUSUM, EWMA

Introduction

In today’s enterprises, a way to increase efficiency, reduce costs and become more competitive is to adopt Total Quality Management (TQM), which aims to enhance quality in all enterprise activities. Statistical Quality Control (SQC) has an important role in the success of TQM.

SQC is the use of statistical principles and techniques at all stages of production to produce a product in the most economical and useful way, to ensure its compliance with predetermined quality specifications and standards, and to minimize the likelihood of defective products [Akin 1996]. The aim is to find the flaws in the enterprise and to take measures by intervening before a defective product is produced [Colak 2007]. Following improvement of the process using statistical methods, an appreciable increase in product quality,
significant decline in error and loss rates, a decrease in production costs, and hence more efficient production are achieved.

The purpose of implementing a quality audit is to monitor the process in the production line, that is, while it is functioning, because it is almost impossible to measure the characteristics of each unit produced. Instead, small samples are taken, measurements are performed, the change over time is presented in drawings, and inferences on the behavior of the process are made. The importance of statistical concepts in quality management arises from the understanding of variability and the probability of realization. The most important tool in understanding and interpreting the variability in the production process and the probability of realization is SQC charts [Ozdamar 2006]. SQC charts were first considered by Shewhart [1924]. An SQC chart is one of the commonly used tools to monitor quality characteristics of interest in a manufacturing process; in other words, to investigate whether or not a process is under control [Haq and Al-Omari 2015].

A review of existing studies of control charts shows that different studies have been conducted in different areas. For example, Gavcar and Aytekin [1995], Sahin [2000], Maness et al. [2003], Pekmezci [2005], Degerli [2006], Ozdamar [2006], Ozdamar [2007], Gedik and Akyuz [2007], Beytekin [2010], Donmez [2012], Franco et al. [2014], and Nazir et al. [2014] have studied Shewhart control charts in different fields.

The Cumulative Sum (CUSUM) control charts developed by Page [1954] as an alternative to the Shewhart control charts began to find widespread use, because they detect small shifts in the process faster. Then, Barnard [1959] introduced the concept of the V-mask, which is effective as an important SQC tool in CUSUM charts. It is seen that CUSUM control charts are applied by different researchers in various fields [Kemp 1961; Harrison and Davies 1964; Williams et al. 1992; Reynolds and Stoumbos 2000; Oktay and Ozcomak 2001; Scandol 2003; Yi et al. 2006; Barratt et al. 2007; Milota 2009; Chan et al. 2010; Castagliola and Malavelakis 2011; Chelani 2011; Riaz et al. 2011; Xia et al. 2011; Yontay 2011; Abbasi et al. 2012; Maravelakis 2012; Cheng and Yu 2013; Mahmoud and Maravelakis 2013; Silvan et al. 2013; Shin and Hwang 2017].

Another alternative to the Shewhart control charts is the Exponential Weighted Moving Averages (EWMA) control charts developed by Roberts [1959] and used as widely as the CUSUM charts. In general, there have been noteworthy studies on EWMA control charts by researchers such as Johnston [1993], Young and Winistorfer [2001], Aydin [2002], Aparisi and Diaz [2007], Serel and Moskowitz [2008], Serel [2009], Ai et al. [2011], Zhang et al. [2014], Azam et al. [2015], Haq et al. [2015], and Raza et al. [2015].

Particularly in terms of quantities of particleboard and fiberboard (MDF) production, Turkey ranks first in Europe and the world, and the wood-based board sector is constantly improving and investments in the sector are increasing day by day. With increasing production and export volumes, quality control
becomes more important at every stage of the process, and many new investments are being made in this field [Istek et al. 2017].

Tensile strength is one of the characteristics that is constantly checked in all particleboard factories as part of quality control in production. Factors such as the method used, chip geometry and amount of glue affect the tensile strength, along with other mechanical properties. During production, the measured tensile strength has a significant effect on the determination of the most appropriate values for these factors [Guler 2015; Istek et al. 2019; Istek et al. 2020]. However, continuous control of mechanical properties such as tensile strength is an important first step in engineering design, especially in areas such as furniture production where particleboards are widely used. These measurements help to observe unexpected strength defects in the products beforehand, and can thus give an idea about actual conditions of usage and damage [Semple and Smith 2006; Efe and Kasal 2007].

In this study, the aim is to contribute to the quality control and quality development activities which constitute an important part of the production costs of enterprises, to minimize quality-related costs, and to ensure the more effective use of operational resources. In this context, control diagrams were prepared to determine and eliminate the factors causing poor quality in terms of the tensile strength values of particleboards.

**Materials and method**

In this study, to implement the necessary statistical techniques, a Turkish particleboard plant engaged in the production of raw and melamine-coated particleboard was selected. The study material consisted of the tensile strength values of boards taken from the plant over three months, from February to April 2016. In the specified date range, the facility underwent one complete maintenance, in the first week of April.

The first stage of the study was the process of obtaining data. For this purpose, samples were taken from the factory, which operated in three shifts per day for three months, following a sampling plan determined by the company. The boards from which the samples were taken were 18 mm in thickness, 630 kg/m$^3$ in density and 2100 × 2800 mm in size, and were produced for internal applications (including furniture) under dry conditions; the sizes and parts of the samples used for the tests were determined following the TS EN 319 [1999] standard. Experimental samples were taken on a total of 6 boards (full length) per day for 42 different days, with at least 10 test pieces from each board. In this study, which was carried out in a commercial enterprise producing boards in accordance with European norms, sampling and conditioning processes were carried out by expert quality engineers in accordance with the standards. In this context, all test samples taken from the boards chosen among those in daily production were conditioned in a conditioning cabinet at 65 ±5%
relative humidity and at 20 ±2°C until they reached a constant mass. Thus, the samples were quickly made ready for the experiments. After the obtaining of samples, Shewhart, CUSUM and EWMA control charts were constructed and evaluated to improve the quality of the production process. MINITAB software was used to create control charts and perform statistical tests.

In the standard Shewhart control charts for the selected samples, arithmetic mean and range graphs were used together, and it was investigated whether any quality characteristics exceeded the control limits. The V-mask procedure was applied in the CUSUM control graphs used to detect small shifts in the mechanical properties of the boards, and 0.5σ and 1σ shifts in the process mean were determined. For this purpose, mean values were found for each sample size of 42 sampling periods, and cumulative totals of difference in target value deviations from sample values were determined. Then, these values were marked on the V-mask to investigate the positive and negative shifts from the target value in the process. A computer program was used for the preparation of the V-mask; nevertheless, the angle θ between the V-mask centerline and the arm was described separately for each deviation. EWMA diagrams, which are another alternative to Shewhart and CUSUM charts and are used to determine the desired level of the process at the desired time and to detect small deviations in the process average, were also created.

**Shewhart, CUSUM and EWMA control charts**

**Shewhart control charts**

Shewhart control charts are the main tools of Statistical Quality Control. These graphs are useful both for measuring the precision of processes and for identifying transferable causes in industrial processes [Topalidou and Psarakis 2009]. Shewhart control charts are classified under two main headings, quantitative and qualitative [Isigicok 2012]:

1. **Quantitative control charts**
   - Arithmetic mean ($\bar{x}$) and range (R) control charts
   - Arithmetic mean ($\bar{x}$) and standard deviation (s) control charts

2. **Qualitative control charts**
   - $p$ control chart
   - $np$ control chart
   - $c$ control chart
   - $u$ control chart

In the study, quantitative control charts were used for measurable properties. Which of the quantitative control charts would be used was determined by the sample size. It is recommended to use ($\bar{x} - R$) control graphs if the sample size is less than 10, while ($\bar{x} - s$) control graphs are recommended if the
Which control chart is the best for the particleboard industry: Shewhart, CUSUM or EWMA?

Sample size is 10 or over [Montgomery 1991]. Therefore, arithmetic mean ($\overline{x}$) and range ($R$) control charts were used.

$\bar{x}$–$R$ control charts

Suppose that $m$ samples are available, each containing $n$ observations of the quality characteristics. Let $(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m)$ be the average of each sample. The grand average of the process $(\overline{\bar{x}})$ is

$$\overline{\bar{x}} = \overline{\frac{\sum \overline{x}_i}{m}} = \frac{x_1 + x_2 + \ldots + x_m}{m} \tag{1}$$

Thus, $\overline{\bar{x}}$ is used as the centerline on the chart. If $x_1, x_2, \ldots, x_m$ is a sample of size $n$, then the range of the sample is the difference between the largest and smallest observations, that is

$$R = x_{\text{max}} - x_{\text{min}} \tag{2}$$

Let $R_1, R_2, \ldots, R_m$ be the range of the $m$ samples. The average range is

$$\overline{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{R_1 + R_2 + \ldots + R_m}{m} \tag{3}$$

The random variable $W = R/\sigma$ is called the relative range. The parameters of the distribution of $W$ are functions of the sample size $n$. The mean of $W$ is $d_2$. Consequently, an estimator of $\sigma$ is $\overline{R} = \overline{R}/d_2$ [Montgomery 2005]. The upper control limits (UCL) and lower control limits (LCL) of the $x$ chart are described as follows [Walpole and Myers 1989; Berenson et al. 1992]:

$$\begin{align*}
UCL &= \overline{x} + A_2 \overline{R} \\
CL &= \overline{x} \\
LCL &= \overline{x} - A_2 \overline{R} \tag{4}
\end{align*}$$

where $A_2$ and $d_2$ are functions of the sample size, and tables are given according to the sample size. The same approach is applied in the construction of $R$ control charts. The centerline and control limits of the $R$ charts are as follows:

$$\begin{align*}
UCL &= \overline{R} D_4 \\
CL &= \overline{R} \\
LCL &= \overline{R} D_3 \tag{5}
\end{align*}$$

where $D_4$ and $D_3$ are functions of the sample size, and tables are given according to the sample size.
CUSUM control charts

The Cumulative Sum (CUSUM) chart was first introduced by Page [1954], and it is a series of sequential operations based on probability ratios to detect the shift in a process [Healy 1987]. Page used CUSUM graphics to determine the defective and non-defective rates of a process and to keep its defective rate under control. Many studies have since been performed to develop CUSUM techniques. In 1959, Barnard introduced the concept of the V-mask, which can be applied to numeric data in CUSUM graphs, and this has been instrumental in making these graphs an SQC tool [Woodward and Goldsmith 1964; Oktay 1994].

CUSUM control charts display the cumulative sum of the deviations of sample averages from a specific target or standard value and the period in the coordinate system. CUSUM control charts are superior to Shewhart’s because they combine information from several samples to determine small process shifts. This advantage is particularly effective in the case of sample volume \( n = 1 \). The structure of the points in the CUSUM graph is more effective than the control limits used in Shewhart graphics. Therefore, the central line and control limits are not calculated in a CUSUM graph [Sarkadi and Vincze 1974; Holmes 1996; Isigicok 2012].

CUSUM control charts are particularly effective in detecting small changes in the process average. Therefore, they have begun to find common usage as standard control charts. However, it is more appropriate to use standard control diagrams in case of large deviations in the process.

The CUSUM chart directly incorporates all of the information in the sequence of sample values by plotting the cumulative sums of the deviations of the sample values from a target value. For example, suppose that samples of size \( n \geq 1 \) are collected, and \( \bar{x}_j \) is the average of the \( j \)th sample. Then if \( \mu_0 \) is the target for the process mean, the cumulative sum control chart is formed by plotting the quantity

\[
S_i = \sum_{j=1}^{i} (\bar{x}_j - \mu_0)
\]  

against the sample number \( i \). \( S_i \) is called the cumulative sum including the \( i \)th sample. Because they combine information from several samples, cumulative sum charts are more effective than Shewhart charts for detecting small process shifts [Montgomery 2005; Kasap 2006]. If the difference \( (\bar{x}_j - \mu_0) \) in the process average shows an increase, the CUSUM graph will show a positive trend, and if the process average decreases gradually, it will show a negative trend [Murdoch 1979].
**V-mask procedure**

In a process, positive and negative shifts from a targeted value may occur. In the CUSUM control graph, the V-mask technique, introduced by Barnard in 1959, was used for the first time to determine whether the shifts in the process average were out of control [Demir and Mirtagioglu 2016].

The V-mask is a method used as a complement to CUSUM graphics to detect out-of-control points. If the cumulative total values in the CUSUM chart are between the arms of the V-mask, it can be said that the process is under control, and if the values are outside the arms of the V-mask, the process is out of control [Oktay and Ozcomak 2001].

The V-mask is applied to successive values of the CUSUM statistics,

\[ S_i = (x_i - \mu_0) + S_{i-1} \quad (7) \]

A typical V-mask is shown in Figure 1 [Montgomery 2005; Kurt 2018].

![Fig. 1. A typical V-mask](image)

In general use, the V-mask is applied to each new point that is graphed. In other words, the mask is rearranged in every new sample. The performance of the V-mask is determined by the distance \( d \) and the angle \( \theta \) [Colak 2007]. The V-mask parameters are calculated according to equation (8) and equation (9) if the value of \( \beta \) is too small to be ignored [Montgomery 2005]:

\[ d = -2 \ln \alpha \frac{\ln \delta}{\delta^2} \quad (8) \]

and

\[ \theta = \tan^{-1} \left( \frac{A}{2A} \right) \quad (9) \]
The definitions of the symbols used in the above equations and in other equations used in the calculation are as follows.

\( \alpha \): The greatest allowable probability of a signal when the process mean is on target (a false alarm).

\( \beta \): The probability of not detecting a shift of size \( \delta \).

\( \Delta \): The shifts in the process average \( \Delta = k \cdot \sigma \).

\( A \): A scale factor, the corresponding value on the vertical axis to a length of 1 unit on the horizontal axis. The value \( A \) varies from \( \sigma \bar{x} \) to \( 2 \sigma \bar{x} \) and it is preferred to take this value as \( 2 \sigma \bar{x} \).

\( \delta \): The smallest amount of shift at the process level to be investigated \( (\Delta = \delta \cdot \sigma) \).

\( \sigma \bar{x} \): Standard error for sample averages \( (\sigma \bar{x} = \frac{\sigma}{\sqrt{n-1}}) \).

\( h \): The value that gives the decision interval when multiplied by the sample statistic \( (H = h \cdot \sigma \bar{x}) \).

\( H \): The decision range of the procedure with the length OU or OL.

\( k \): The value that gives the slope of the V-mask arms when multiplied by the sample statistic \( (K = k \cdot \sigma \bar{x}) \).

\( K \): The slope of the V-mask arms.

\( d \): The value of the length OP.

\( \theta \): The angle between the centerline and the arm [Kartal 1999; Demir 2008].

**EWMA control charts**

EWMA (Exponential Weighted Moving Averages) control charts, which are also called geometric moving average diagrams, were first introduced by Roberts [1959]. Roberts’ studies were followed by those of Hunter [1986], Crowder [1987] and Lucas and Saccucci [1990]. The decision in the EWMA control technique depends on the EWMA statistic, which gives weight to old observations [Testik 1999].

While the performance of the EWMA control chart is very similar to the CUSUM control graph, its creation and implementation are easier than in the CUSUM case. The EWMA control chart can be graphed as easily as Shewhart’s. Therefore, it is a good alternative to Shewhart control diagrams to detect small shifts in the process. In some cases, EWMA can also be used to predict the next observation [Ege 2000].

The EWMA estimation value is calculated as follows:

\[
\begin{align*}
    z_i &= \lambda \bar{x}_i + (1 - \lambda) z_{i-1} \\
    z_{i-1} &= \lambda \bar{x}_{i-1} + (1 - \lambda) z_{i-2}
\end{align*}
\]

(10)

\( 0 < \lambda < 1 \) is a constant, and the starting value (required with the first sample at \( i = 1 \)) is the process target. As this value gets closer to 1, the weight of the last observation value increases; as it gets closer to 0, the weights of the old
Which control chart is the best for the particleboard industry: Shewhart, CUSUM or EWMA?

Observation values increase. In general, values of $\lambda$ in the interval $0.05 < \lambda < 0.25$ work well in practice, with $\lambda = 0.005-0.10-0.20$ being popular choices in the literature. A good rule of thumb is to use smaller values of $\lambda$ to detect smaller shifts [Montgomery 2005].

The control limits of the EWMA control chart are as follows:

$$UCL = \bar{x} + L \sigma \sqrt{\frac{\lambda}{2 - \lambda}} \left[ 1 - \left( 1 - \lambda \right)^{2i} \right]$$

$$CL = \bar{x}$$

$$LCL = \bar{x} - L \sigma \sqrt{\frac{\lambda}{2 - \lambda}} \left[ 1 - \left( 1 - \lambda \right)^{2i} \right]$$

where $L$ is the size of the control limits, and this seems to work well at the commonly used $3\sigma$ levels [Montgomery 2005].

Results

Shewhart control chart results

To set up arithmetic mean and range graphs of Shewhart control charts, the central values of $\bar{x}$ and $R$ graphs were found. The central value of the $R$ graph was obtained as follows:

$$\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{4.187377}{42} = 0.0997$$

Since the sample sizes were 6, the table values of $D_3$ and $D_4$ were determined as $D_3 = 0$, $D_4 = 2.004$. Thus, the control limits of the $R$ graph were obtained as follows:

$$UCL = \bar{R} D_4 = 0.0997 \cdot 2.004 = 0.1998$$

$$CL = \bar{R} = 0.0997$$

$$LCL = \bar{R} D_3 = 0.0997 \cdot 0 = 0$$

The central value of the graph of $\bar{x}$ was

$$\bar{x} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = \frac{18.46233}{42} = 0.439579$$

The $A_2$ table value was 0.483, and the control limits of the $\bar{x}$ graph were obtained as follows:

$$UCL = \bar{x} + A_2 \bar{R} = 0.439579 + 0.483 \cdot 0.0997 = 0.4878$$

$$CL = \bar{x} = 0.439579$$

$$LCL = \bar{x} - A_2 \bar{R} = 0.439579 - 0.483 \cdot 0.0997 = 0 = 0.3914$$
In Figure 2, \( \bar{x} \) and \( R \) control graphs of tensile strength values are given. When the graph is examined, it is seen that the \( \bar{x} \) graph gives an out-of-control signal. Furthermore, the values are not only distributed on the middle line, but they also display an unstable trend. If the points 7, 24 and 25 on the \( \bar{x} \) graph exceed the lower control limits, the process is out of control. Similarly, it is seen that values of 34, 35, 37 and 38 exceed the upper control limits in the \( \bar{x} \) graph. Since this is seen as a desirable result for the production facility, no consideration has been given to it. When the \( R \) graph is examined, it is seen that there is no value overflowing out of control; the measured values fluctuate around the central value.

**CUSUM control chart results**

To determine whether the CUSUM values were out of control, the parameters of the V-mask to be plotted were found. The V-mask was drawn utilizing a computer program based on the parameters \( h \), which is a measure of the decision range, and \( k \), which is known as the reference value. The values of \( h \) and \( k \) were taken as 4 and \( \frac{1}{2} \), respectively, due to the widespread use of these values in the literature and their high ARL performances. CUSUM control graphs showing 0.5\( \sigma \) and 1\( \sigma \) separations of the tensile strength values of the boards were prepared.

In Table 1, measurement values of tensile strength, sample means, deviations from the sample means and cumulative CUSUM values calculated according to these values are given.

The parameters of the CUSUM control graph can be calculated using equations (6) and (7). Below is the calculation of \( S_i \) values for the first two samples.
Which control chart is the best for the particleboard industry: Shewhart, CUSUM or EWMA?

Table 1. CUSUM values calculated for tensile strength

<table>
<thead>
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<th>No</th>
<th>$\bar{x}_i$</th>
<th>$\bar{x}_i - \bar{x}$</th>
<th>$S_i$</th>
<th>No</th>
<th>$\bar{x}_i$</th>
<th>$\bar{x}_i - \bar{x}$</th>
<th>$S_i$</th>
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</tr>
</tbody>
</table>

\[ \bar{x} = 0.439579 \]

\[
S_1 = \sum_{i=1}^{j} (0.399624 - 0.439575) = -0.03996 \\
S_2 = (0.427306 - 0.439575) + (-0.03996) = -0.05223
\]

If the standard deviation of \( \bar{x}_i \) is indicated by \( \sigma_{\bar{x}} \), if \( \alpha \) and \( \beta \) are kept at the levels \( \alpha = 0.01 \) and \( \beta = 0.01 \) as commonly used in the literature, and \( d \) is taken as \( d_2 = 2.534 \) with the help of the table, the parameters giving the slope of the V-mask arms are found by the following equations:

\[
\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{4.187377}{42} = 0.0997 \\
\alpha = \frac{\bar{R}}{d_2} = \frac{0.0997}{2.534} = 0.03934
\]

To find \( 0.5\sigma \) shifts, the value \( \Delta \) is calculated as follows:

\[ \Delta = k \sigma = 0.5 \cdot (0.03934) = 0.0197 \]
To determine the length $d$ between the end of the V-mask and the sample point to be placed on the V-mask, the values $\sigma_x$ and $\delta^2$ must be found. Thus, the standard error of the distribution is

$$\sigma_x = \frac{\sigma}{\sqrt{n-1}} = \frac{0.03934}{\sqrt{5}} = 0.01759$$

The value of $\delta^2$ is obtained as follows:

$$\delta^2 = \left( \frac{A}{\sigma_x} \right)^2 = \left( \frac{0.0197}{0.01759} \right)^2 = 1.254$$

The length $d$ is

$$d = -2 \frac{\ln \alpha}{\delta^2} = -2 \frac{\ln 0.01}{1.254} = 7.345$$

Also required is the angle $\theta$ made by the V-mask with the horizontal axis. To find this angle, in addition to the information above, $A$ must be known. The value of $A$ is found as follows:

$$A = 2 \sigma_x = 2 \times (0.01759) = 0.03519$$

Thus, the angle $\theta$ can be determined as follows:

$$\theta = \tan^{-1} \left( \frac{A}{2A} \right) = \tan^{-1} \left( \frac{0.0197}{2 \times (0.03519)} \right) = 15.637^\circ$$

This shows the angle that the V-mask makes with the length $d$ of the upper and lower arms.

In Figure 3a, the V-mask CUSUM graph is presented, indicating that the 0.5$\sigma$ separations of the tensile strength are out of control. When the graph is examined, it is seen that the CUSUM control chart shows a negative trend, that is, the process average decreases and then resets. Moreover, when the V-mask is placed on sample 42, it is found that samples 19 and 24-37 are out of control. This means that the process average has changed over time. The control graph in Figure 3b was obtained by placing the V-mask on the sample giving the first out-of-control signal, to determine after which sample the process average changed or went out of control. When the V-mask is placed on sample 7 (Fig. 3b), it is seen that the process is out of control from the first sample, that is, the process average changed starting from the first sample.

To find 1$\sigma$ shifts in the process average, the value $\Delta$ was calculated as follows:

$$\Delta = k \sigma = 1 \times (0.03934) = 0.03934$$

Since the standard error of the distribution was set to 0.01759 in the previous V-mask, $\delta^2$ was found as follows:

$$\delta^2 = \left( \frac{A}{\sigma_x} \right)^2 = \left( \frac{0.03934}{0.01759} \right)^2 = 5$$
a. CUSUM control chart detecting the $0.5\sigma$ shifts

**Fig. 3. V-mask graphics for $0.5\sigma$ shifts**

The length $d$ is

$$d = -2 \frac{\ln \alpha}{\delta^2} = -2 \frac{\ln 0.01}{5} = 1.842$$

Since the value $A$ was calculated as 0.03519 in the previous calculation, the angle $\theta$ between the V-mask arms and the length $d$ was obtained as follows:

$$\theta = \tan^{-1} \left( \frac{A}{2A} \right) = \tan^{-1} \left( \frac{0.03934}{2 \cdot 0.03519} \right) = 29.20^\circ$$

Figure 4a shows the V-mask prepared to find $1\sigma$ separations in the tensile strength values. When the graph is analyzed, it is seen that the $1\sigma$ signal is out of control. Samples 31-34 were out of control when the V-mask was placed on the last sample taken. To determine from which sample the process average changes,

a. CUSUM control chart detecting the $1\sigma$ shifts

**Fig. 4. V-mask graphics for $1\sigma$ shifts**

b. CUSUM control chart obtained by placing the V mask on the sample that gives the first out of control signal (for $1\sigma$ shifts)
the V-mask is placed in the sample giving the first out-of-control signal (Fig. 4b); it is seen that sample 33 is out of control. This shows that the process average changed from sample 33. At the same time, the outside point is located outside the lower arm of the V-mask, indicating that the process average tends to shift upwards. That is to say, there is an increase in average tensile strength values starting from sample 33.

**EWMA control chart results**

In this study, the most suitable \( \lambda \) value was found to be 0.1, and the \( L \) value was found as 3 in determining the EWMA values of tensile strength. For the first sample mean (0.399624), the EWMA value was determined as follows:

\[
z_i = \lambda x_i + (1 - \lambda) z_{i-1} = 0.1 \cdot (0.399624) + (1 - 0.1) \cdot (0.439579) = 0.43558
\]

The first point, 0.43558, was marked on the EWMA control plot. The EWMA value to be calculated for the second sample average (0.427306) was obtained as follows:

\[
z_2 = \lambda x_2 + (1 - \lambda) z_1 = 0.1 \cdot (0.427306) + (1 - 0.1) \cdot (0.43558) = 0.43475
\]

For all the remaining EWMA values \( i = 1 \) to 42, the calculations were performed similarly and processed on the graph. In the study, these values were calculated only for the first two samples. Other calculation results are given in Table 2.

After the determination of EWMA values, equation (11) was used to determine the control limits of these values. Using the formula, the UCL value at \( i = 1 \) is

\[
UCL = 0.439579 + 3 \cdot (0.03934) \sqrt{\frac{0.1}{(2-0.1)}} \left[ 1 - (1 - 0.1)^{2(1)} \right] = 0.451381
\]

The LCL value at the same time is

\[
LCL = 0.439579 - 3 \cdot (0.03934) \sqrt{\frac{0.1}{(2-0.1)}} \left[ 1 - (1 - 0.1)^{2(1)} \right] = 0.427777
\]

The control limits of the second sample are given below:

\[
UCL = 0.439579 + 3 \cdot (0.03934) \sqrt{\frac{0.1}{(2-0.1)}} \left[ 1 - (1 - 0.1)^{2(2)} \right] = 0.455457
\]

\[
ADS = 0.439579 - 3 \cdot (0.03934) \sqrt{\frac{0.1}{(2-0.1)}} \left[ 1 - (1 - 0.1)^{2(2)} \right] = 0.423701
\]

The control limits calculated for other times are presented in Table 2.

By transferring the control limits and EWMA values in Table 2 to the control graph, the EWMA chart of tensile strength in Figure 5 was obtained. In EWMA graphs, the upper and lower control limits generally remain constant after a point, and from this point onwards they take maximum and minimum values.
Which control chart is the best for the particleboard industry: Shewhart, CUSUM or EWMA?

Table 2. EWMA values and EWMA control limits for tensile strength

<table>
<thead>
<tr>
<th>i</th>
<th>( \bar{x} )</th>
<th>EWMA ((z_i))</th>
<th>Control limits</th>
<th>i</th>
<th>( \bar{x} )</th>
<th>EWMA ((z_i))</th>
<th>Control limits</th>
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</table>

When Figure 5 is examined, it is seen that the EWMA values of the tensile strength are within the control limits. However, the exponential weighted moving averages of tensile strength are quite volatile. In particular, it is observed that the tensile strength values decrease and reach the lowest limit at the 8th sample. They start to increase from the 33rd sample and reach the highest value at the 41st sample.
Conclusion and recommendation

In the study, in total, 42 × 6 samples for determination of the mechanical properties of particleboards were taken from a particleboard factory from February to April, and Shewhart, CUSUM and EWMA control diagrams were prepared. The results obtained are summarized below.

When the Shewhart control charts of the tensile strength values were examined, it was found that there were no problems in the $R$ chart because the samples remained within the control limits, and that the samples were mutually compatible. However, in the $x$ chart, samples 7, 24 and 25 exceeded LCL, and samples 34, 35, 37 and 38 exceeded UCL, which indicated that the process mean was not at a constant level and was out of control. Since the tensile strength value exceeding UCL was a desired situation for the production facility, it was not emphasized. The special causes of the values exceeding LCL were investigated with the company’s quality control team, and it was discovered that the low resistance in sample 7 was due to the low-pressure temperature in that period, and the low resistance in the samples numbered 24 and 25 was caused by an error in the injectors in the gluing system.

When the CUSUM tensile strength values were analyzed, it was determined that shifts of $0.5\sigma$ and $1\sigma$ gave out-of-control signals. In the case of $0.5\sigma$ shifts, a total of 15 points were outside the lower arm of the V-mask, which indicated that the process average changed over time. To determine from which sample the process average changed, the V-mask was placed on the sample that gave the first out-of-control signal, and it was revealed that the process average started to decrease from sample 1. Considering the $1\sigma$ shifts, a total of 4 points were out of control. Again, to determine from which sample the process means changed, the V-mask was placed on the sample that gave the first out-of-control signal (sample 37), and it was seen that the process mean changed from sample 33. In other words, the process average, which had decreased until the 33rd sample,
Which control chart is the best for the particleboard industry: Shewhart, CUSUM or EWMA?

started to increase at that point. When the reason for that situation was investigated, the increase was determined to have occurred since the facility’s maintenance period.

When the EWMA control chart of tensile strength was observed, no out-of-control signal was detected. However, the sample averages decreased until sample 33, and samples 7 and 8 were close enough to LCL to attract attention. Also, as in the CUSUM chart, an increase in averages after sample 33 was noticeable in the EWMA chart.

When the results of the control diagrams were surveyed in general, it was inferred that the CUSUM control charts gave very effective results in detecting small changes in the process. Although both the CUSUM and Shewhart control charts indicated that there was an uncontrolled point in the process, the appearance of the CUSUM chart gave a clearer visual picture and showed from which point the shift was present. This will help the business to decide when to perform maintenance and to determine maintenance periods, as with the change after sample 33. CUSUM charts were more effective in providing 0.5σ and 1σ separations and supplied healthier information to the manager when looking at the general condition of the process. When the prepared CUSUM graphics were applied to the last sample, it also provided an advantage to the company by giving information about the past samples. Furthermore, the fact that CUSUM charts are effective even when \( n = 1 \) will enable the business to incur lower costs.

Shewhart control graphics were found to be more successful with large-scale shifts. It was observed that the results were effective only when the samples were taken independently. Besides, it gave better results than other graphs in determining the causes of errors during the period when the sample was taken.

When looking at EWMA control charts, it was seen that they provided ease of application and gave information about the general condition of the process, like CUSUM charts. It was concluded that there may be a problem in the process, and the charts may assist the auditor in deciding whether to intervene. However, it was observed that EWMA control diagrams were not as successful as other control charts in detecting out-of-control points.

In the light of all of these evaluations, it is concluded that using only one of Shewhart, CUSUM and EWMA control charts may be misleading when making decisions about the process status of a production facility, and so using at least two will be more appropriate. Another conclusion is that evaluation of the observation values obtained from the particleboard industry, both interdependent and independent from each other, will yield more effective and efficient results. For this reason, it is suggested to use CUSUM and Shewhart control charts as the most suitable control graphics.

In the literature, similar results are encountered in research on statistical control graphics. In the studies of Oktay [1994] and Demir [2008], who compared Shewhart, CUSUM and EWMA control charts practically, it was
found that they obtained better results with large-scale shifts in the process, and with CUSUM control charts for small-scale shifts. Nenes and Tagaras [2007] also stated that CUSUM control charts were more economically advantageous than $x$ control charts. Milota [2009] stated that, in a study carried out in the timber industry with $x$ and CUSUM control charts, what the $x$ average charts could not detect was easily detected with the CUSUM control charts. In another study, Ozcil [2014] compared Shewhart, CUSUM and EWMA control charts, and suggested the Shewhart as the most appropriate control chart, since she believed that the data in her field of application were evaluated independently.

There are many factors and parameters that affect the mechanical properties of particleboards. However, the unwillingness of Turkish companies to share data is restricting efforts to achieve more efficient and thorough operation. With more variables and test data available from businesses, healthier forecasts and results can be obtained.

Statistical techniques are extremely effective in businesses, both for improving current quality and for determining the future status of a process. By combining these techniques with different methods, they can be easily applied not only in large-scale establishments, but also in small-scale ones, and significant reductions in quality-related costs can be achieved.

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